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23rd Annual Conference of the
Transportation Science Society of the Philippines
Quezon City, Philippines, 8 August 2016

Calibrating Relative Velocity and Lateral Clearance Parameters of a Lane Changing Model for Traffic Microsimulation

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Abstract: Traffic simulation is currently one of the most used and proven effective tool in traffic management. Its ability to emulate vehicles and other associated elements with the corresponding time variability and faculty for interfacing with the surroundings made traffic simulations suitable and distinctly adaptable for evaluation, planning, and analysis of complex transportation systems. Herewith, the Department of Science and Technology (DOST) in the Philippines has proposed a project which aims to develop a local traffic simulation which will be initially implemented in EDSA. Nevertheless, the lane changing model, which is a subcomponent of the local simulations, is currently inadequate of several threshold parameters which is the lateral clearance maintained between vehicles and the limit velocity difference. Consequently, the study intends to determine the parameters required for the lane changing model through video surveillance with the aid of various statistical methods. Scatter plots and frequency distributions of the parameters were generated for different types of vehicles.

Key words: lane changing, lane changing parameter, traffic microsimulation

1. INTRODUCTION

1.1 Traffic Simulation

The increasing advancement in the field of computer technology, engineering innovations, and software applications has revolutionized the way of approach in the field of urban planning and traffic management. This rapid progression of technology has impelled the advent of traffic simulation which is nowadays has been one of the most effective tool in facilitating transportation systems such as arterials, freeways, and interchange.

Traffic simulation is a process of replicating or imitating transportation systems mathematically through the use of actual field parameters incorporated in a certain simulator. Furthermore, traffic simulation can be scrutinized into various approaches. For instance, vehicles can be viewed as a group of individuals or entities travelling within a system. This category of approach is labeled as Macroscopic. Another method of simulation is the Microscopic approach wherein a vehicle's individual behavior are examined and analyzed. In the microscopic approach, every individual detail of vehicles is modeled with the purpose of creating a unique entity capable of interaction with other elements or subject components in the model. There are numerous traffic micro-simulation models developed and each of those models involve various traffic interactions with different interface such as car following, lane changing, bus stops, pedestrian movements, signalized and unsignalized intersections, spatial collision, etc. Car Following Model is one of the established microscopic models and has been studied for the past 50 years. This model involves a vehicle following another vehicle by assessing the relative distances, speed differences, and reaction time of the subject vehicles. Another recognized component of microscopic model is the Lane Changing Model in which a vehicle transfers lane depending on the velocity, acceleration, distance, politeness factor, etc. of the lane changing vehicle in comparison with the adjacent vehicles. (Trani, Introduction to Transportation Engineering, 2009).

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Traffic Simulation has been a vital facet of traffic management. Through the emulations of the dynamics and behaviors of vehicles, the effectiveness of local traffic systems can be vastly improved. Moreover, these emulation-based simulations can be further used in different transportation design and operations.

1.2 Lane Changing Model for a Local Traffic Simulator

Effort is on-going to develop a localized microscopic traffic simulation software with the end goal of piloting simulation of traffic flow in EDSA as a part of a research project entitled "Customized Local Traffic Simulation (LOCALSIM)" funded by DOST-PCIEERD. However, several vehicle behavior models, particularly lane changing, is still in the process of development and requires estimation of parameters for calibration.

The lane changing model of the local traffic simulator incorporates the concept of optimizing the speed of the subject vehicle at the next time step. It incorporates the MOBIL (Minimizing Overall Braking Induced by Lane Changing) and identifies threshold relative velocity and limit lateral clearance as two important parameters to calibrate the behavioral model. Fundamentally, the limit relative velocity is the threshold discrepancy in velocity, with respect to the lead car, that a lane changing vehicle can endure before deciding to look for a more favorable lane while the limit lateral clearance pertains to the lane changing vehicle's threshold lateral gap or clearance needed before attempting to transfer lanes. Figure 1 and Figure 2 illustrates a typical scheme in the MOBIL lane changing model with the corresponding subject parameters.
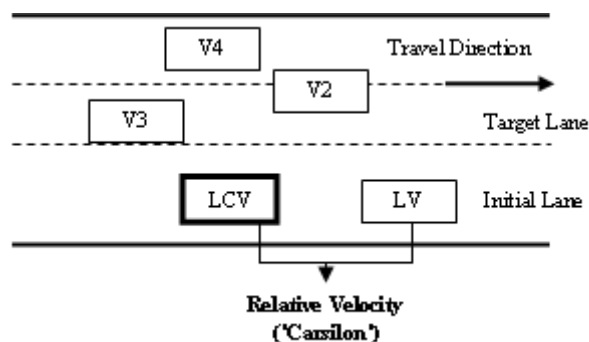

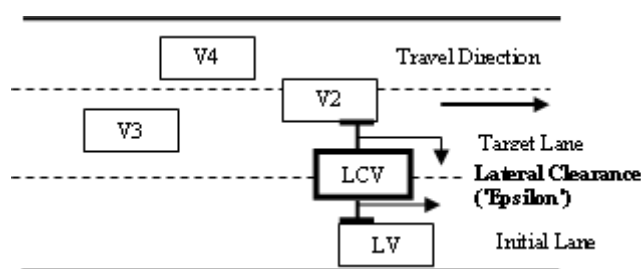Figure 1: Lane changing (Initial time step)

Figure 2: Lane changing (Subsequent time step)

Threshold relative velocity and threshold relative clearance are the key parameters for the instigation of lane changing. Figure 3 illustrates the process by how a vehicle decides whether to lane change or not. It is explicitly shown in Figure 3 that as the lane changing vehicle (LCV) perceived the lead vehicle (LV), LCV will asses if LV is too slow. Herewith, if the velocity difference of LV and LCV exceeds a certain limit (Carsilon), the subject vehicle will seek for a more convenient lane. After selecting the target lane, the vehicle will then gauge whether its width plus some threshold lateral allowance (Epsilon) will fit into the desired lane.
Since Filipino drivers were used to cut in any possible space available regardless of alignment with lanes, as shown by adjacent vehicle 2 (V2), the space available for the possible target lane will likely reduce. Thus, it would be essential to consider Epsilon to make the model as much as

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possible close to the actual Philippines setting. In other countries, only the longitudinal gap or the clearance between LCV and LV was considered instead of the lateral allowance (Epsilon).
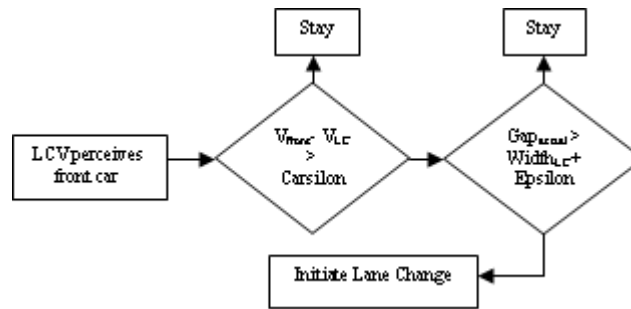

Figure 3. Initiation of Lane Changing

1.3 Study Objective

The local simulation being developed in the Philippines are still in need of several threshold parameters which will correspond and characterize how Filipino drivers change lanes. These parameters, particularly lateral clareance limit ('Epsilon') and velocity difference limit ('Carsilon'), are the main subject of the study.

The study aims to determine the threshold values for the parameters lateral clearance and relative velocity of lane changing vehicles in EDSA which will be executed by capturing videos of vehicles shifting lanes with the aid of various statistical methods and several scaling softwares. These parameters will be used to calibrate the local traffic simulation being developed by DOST.

The study will provide the required parameters for Lane Changing model. These parameters would be a vital component in the assembly of Local Traffic Simulations for EDSA. Upon completion of the local simulations and modeling, traffic congestion in EDSA is expected to be facilitated. Likewise, car accidents are likely to be reduced. Also, an enhanced and a more developed system for traffic flow and safety can be devised from the completed local traffic micro-simulations. Traffic simulations can further be used for network design, evaluation, planning and analysis. But above and beyond all considerations, the primary asset of developing traffic simulations is the opportunity to immediately observe what would be the outcome or consequence of a certain improvised traffic strategy or devised alternative schemes based on a simulated model before implementing it on the actual field. In this manner, proposed traffic schemes will be evaluated beforehand prior to execution thus circumventing unnecessary costs due to an ineffective project.

1.4 Scope and Limitation

Data collected were extracted from video footage wherein the camera is placed at an elevation of about 7 m only. The data would be more accurate if it will be taken at a higher elevation. Generally, trajectory data are taken from a helicopter or a drone to capture wider view of the subject vehicles. With this kind of setup, it will allow the camera's plane of view to be parallel to the top of the subject vehicles. But with limited elevation, the plane of view of the camera is forced to be slightly angled from the ground for it to capture a wider view of the subject vehicles. Furthermore, parameters that will be obtained in the study may not be applicable in other locations but EDSA. Also, the simulations that will be designed from the obtained parameters are for a one-way multilane freeway only. Opposing travel direction in the target lane will not be considered.

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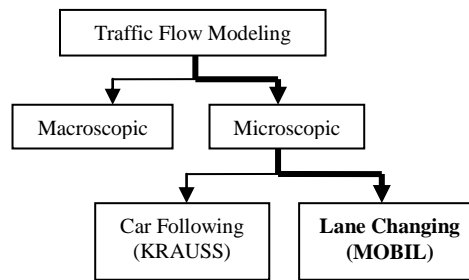23rd Annual Conference of the
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1.5 Conceptual Framework

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            ┌─────────────────────┐
            │ Traffic Flow Modeling│
            └─────────────────────┘
              │                 │
              ▼                 ▼
        ┌───────────┐     ┌───────────┐
        │ Macroscopic│     │ Microscopic│
        └───────────┘     └───────────┘
                           │          │
                           ▼          ▼
                    ┌───────────┐  ┌───────────┐
                    │Car Following│  │Lane Changing│
                    │  (KRAUSS)  │  │  (MOBIL)   │
                    └───────────┘  └───────────┘
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Figure 4. Underlying Concepts of Traffic Flow Simulation

2. REVIEW OF RELATED LITERATURE

2.1 Multi-Parameter Prediction of Drivers' Lane Changing Behaviour with Neural Network Model – Jinshuan Peng, Yingshi Guo, Riu Fu, Wei Yuan, Chang Wang

During lane changing process, the workload of the driver significantly increases. Performing the manuever will require the driver to assess the lead car velocity and its longitudinal gap. The driver will later evaluate target lane accessibility ensuring that obstructions are clear. Moreover, failure to properly gauge relative movement of adjacent vehicles may lead to damaging properties and casualties (Petzold et al., 2014; Jin 2013).

In recent years, lane change auxiliary systems have been developed to mitigate the problem on the subject of lane changing. The system operates through the help of a milimeter-wave radar and high accuracy cameras. As the auxiliary systems perceive incoming obstructions, the driver will be provided with signals indicating probable danger (Hirose et al., 2004).

2.2 Lane Changing Models for Arterial Traffic – Varun Ramanujam

Lane changing process is further classified into categories explicitly Mandatory Lane Changing (MLC) and Discretionary Lane Changing (DLC) depending on the intent or motive that initiate the lane changing. A lane change is mandatory if it is obligatory to transfer lane due to an unforeseen circumstance, approaching obstruction, or exit junction. In contrast, a discretionary lane change occurs when the vehicle search for a lane with better driving conditions than its initial lane.

Yang and Kuotsopoulus (1996) modelled driving behaviour wherein MLC is instigated by a specific probability that varies along with several factors covering traffic density, distances from an exit point, etc. On the other hand, DLC is initiated when the desired velocity of the vehicle is not attained. Furthermore, Ahmed (1999) designed a more detailed framework of a lane changing decision process with its components MLC and DLC.
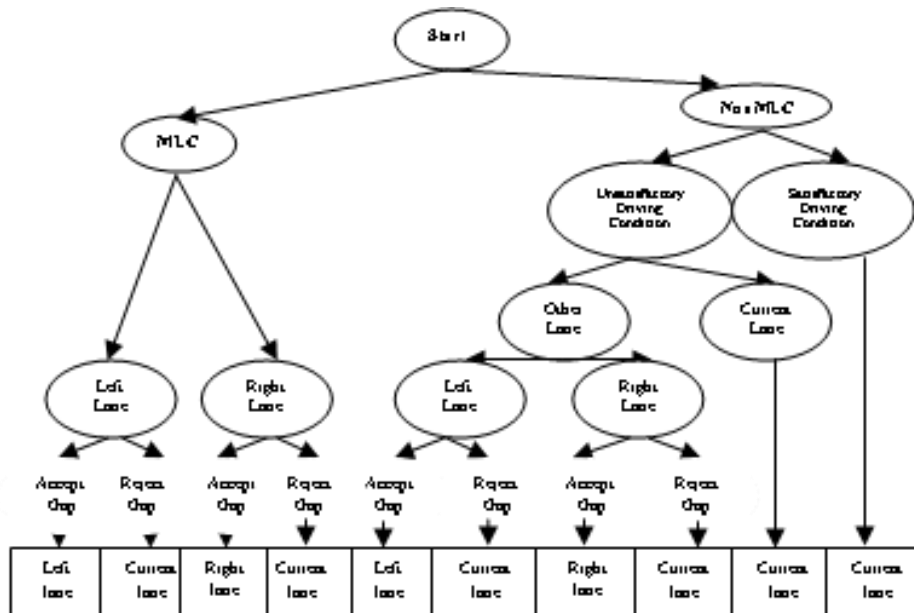
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Figure 5. Detailed Framework of Lane Changing Decision Process
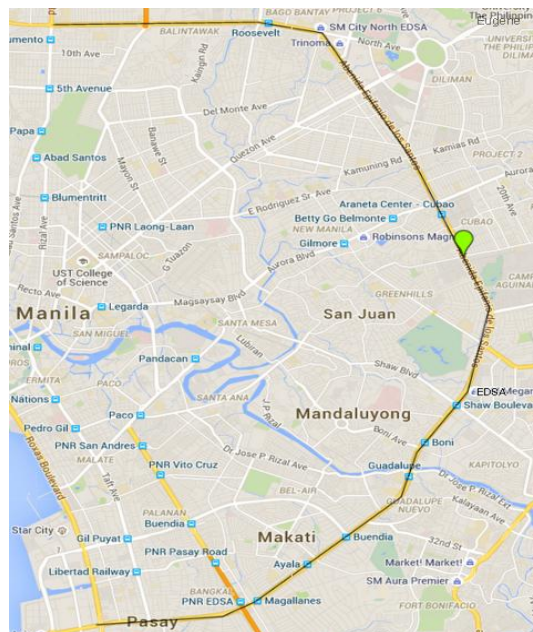(Source: Lane Changing Models for Arterial Traffic-Varun Ramanujam)

3. METHODOLOGY

Figure 6. Study Area

3.1 Site Selection

The collection of data is situated along Epifanio de los Santos Avenue (EDSA) in Metro Manila. EDSA, as commonly referred by many, is a limited access freeway which is a part of a network of routes linking North Luzon Expressway within Balintawak to South Luzon Expressway in Magallanes junction. The said road extends approximately 23.8 km (14.8 mi) and is consisted of 6 lanes per direction. It has been estimated that almost 2.34 million vehicles travel EDSA every day thus becoming the most congested highway in Metro Manila. For this reason, traffic

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simulation would be essential to alleviate traffic obstructions and to further improve schemes for traffic safety within the road.

3.2 Sampling

The data were gathered specifically at Cubao, Ortigas, and Quezon Avenue in EDSA. Since the parameters are variables of kinematics, location of survey will not induce an effect to the results. Furthermore, 100 data points per vehicle type (Cars, SUVs & Vans, Buses and Trucks) were used for the computation of the parameters. Vehicles that will be a part of the sample are the auto mobiles that are able to perform lane changing manuever completely. Nevertheless, auto mobiles which are wedged in between two lanes, not able to complete the manuever, due to unexpected obstruction in the target lane will not be included.

3.3 Experimental Setup

From a high vantage point, surveillance camera will be installed to record vehicles as it changes lanes. However, due to the limited elevation of the place where the camera can be mounted, the video camera was forced to be slight angled from the ground to capture a wider view of the traffic flow. As a way to compensate this limitation, only the vehicles near the setup camera were measured to minimize parallax error. Also, in measuring for the parameters, grid lines were drawn aligning the horizontal lines with the pavement markings and the transverse line perpendicular to the broken pavement markings. This gridlines are essential and critical for measuring since it will be used for the scaling.

3.4 Data Collection

Data collection will be conducted for one week from 7 am to 8 pm to account all traffic conditions (Free flow condition and Congested). The device that will be used for capturing video footages is Gopro.

3.5 Analysis

Through the use of scaling softwares such as Brava!, the videos obtained from the field survey can be converted into frames per second enabling to measure the distances and velocities required for the computation of parameters. For the measurement of the velocity difference of the subject vehicle and the lead vehicle, several preliminary considerations must be taken into account. The length of the white broken pavement parking is known to be about 3 m and the spacing in between these markings is 6 m. These established values will be used as a basis for scaling for distances covered by vehicle at a certain time which will later yield its velocity. For the measurement of the lateral clearance, a measured value of 3.5 m will be utilized as the width of each lane.

Central tendency calculations, probability distributions, and other statistical tools will aid in obtaining the values for the parameters required. Nonetheless, it is not necessary that the parameters would be a discrete value. The parameters may be a function of a certain data or variable. Essentially, the parameters may be incorporated into a previously developed lane changing model and later compare it to the outcome yielded. After ensuring the accuracy of results, the parameters specifically threshold lateral clearance (Epsilon) and threshold relative velocity (Carsilon) can already be applied to the traffic simulations of DOST. The schematic diagram below illustrates the summary of procedures in the analysis of data.

4. Results and Discussion

4.1 Threshold Lateral Clearance

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After several measurements of the parameters, a series of plots were generated relating the speed of the vehicles and the lateral clearance of the interacting vehicles. In Figure 6a, the red dots pertain to the lateral clearance from left side of the vehicle while the blue dots pertain to the right lateral clearance. Furthermore, Firgure 6b was the plot obtained when the minimum value between the left and right lateral were taken and Figure 6c was the plot generated if the average of the left and right lateral clearance were taken. It can be observed from the three plots there is significant relationship between the speed of the vehicle and their maintained lateral clearance. As the subject vehicle's speed increases, the sustained lateral clearance also increases. The correlation between the two discussed parameters appears to be linear.
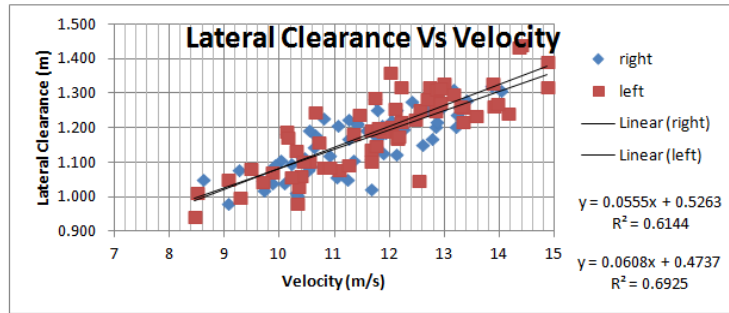
Figure 6a. Lateral Clearance in Relation with Velocity
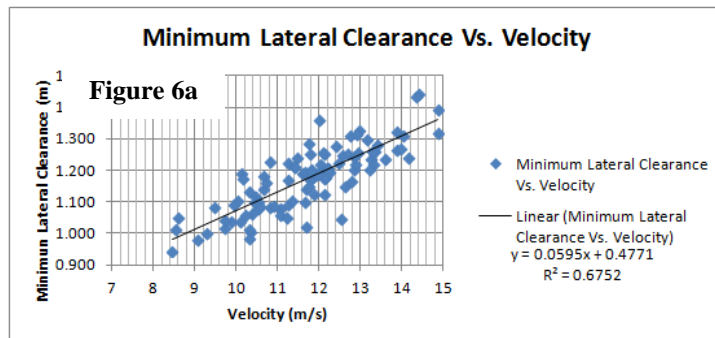
Figure 6b. Minimum Lateral Clearance in Relation with Velocity
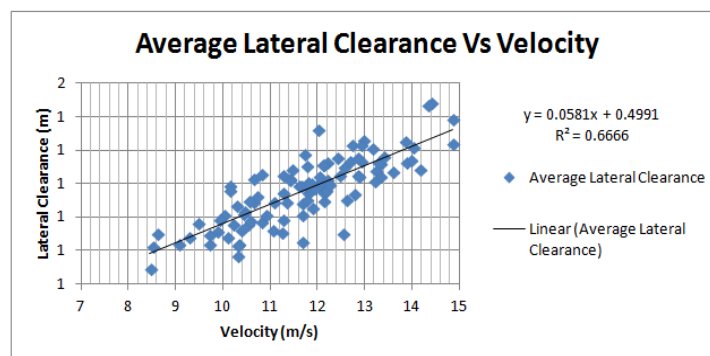
Figure 6c. Average Lateral Clearance in Relation with Velocity

4.1.1 Lateral Clearance per Vehicle Type

From the previous plots, all types of vehicles were used for the measurement of the required parameters which means that their physical dimensions are not constant. This variation especially in terms of width may have a significant effect on the values of parameters measured. To further improve the accuracy of the data, the lateral clearances were also computed per vehicle

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types which were categorized into Cars, SUVs & Vans, and Buses & Trucks. The plots of the lateral clearance per vehicle type were shown in sections 4.1.1.1 to 4.1.1.3. A summary of the values from the regression fit and central tendency was provided in Table 1 which includes the mean velocity, mean left and right lateral clearance, standard deviation, slope of the linear equation, and the correlation factor. Based on the Table 1, it can be observed that the correlation factor R^2 improved as the parameters were separated per vehicle type due to the minimized deviation of physical dimension of the subject vehicles. Furthermore, it can also be inferred from the table the changes in the mean velocities and lateral gaps of each vehicle types. Cars tend to maintain the largest lateral clearance since they travelled at a faster pace. On the other hand, Buses and Trucks travelled in a slow pace which only requires a small lateral clearance from another vehicle. On the contrary, its slope suggests a different perspective. Buses and Trucks have the highest slope compared to the other vehicular categories which implies that, for a certain speed, they tend to set a larger lateral clearance. To comply with this contradiction, it is plausible to assume that most of the buses and trucks travel in a slower pace though they sustain a slightly larger transverse gap for a given speed. Although in terms of risk factor, the slope would suggest that Buses and Trucks are more cautious, several factors must still be evaluated before considering the preliminary observation. Even if the bus maintain a lateral clearance that is safe enough for its speed, that lateral clearance may not be enough if the speed of its adjacent vehicle travels at a faster rate. Also, larger vehicles must have an additional lateral clearance to compensate its weight. With this, the speed and type of the adjacent vehicles should be studied and included in the overall safety of traffic flow.

Table 1. Lateral Clearance per Vehicle Type

Vehicle Type	Mean Velocity (m/s)	Mean Lateral Gap (L) (m)	Mean Lateral Gap (R) (m)	Slope (L)	Slope (R)	R^2 (L)	R^2 (R)
Without Separation	11.716	1.193	1.16	0.0556	0.0596	0.6409	0.6784
Cars	11.851	1.181	1.85	0.053	0.0548	0.6935	0.7132
SUVs & Vans	11.595	1.152	1.156	0.589	0.0628	0.6847	0.7539
Buses and Trucks	10.303	1.063	1	0.0771	0.0777	0.8297	0.7315

4.1.1.1 Cars

Figure 7a. Lateral Clearance in relation with Velocity (Cars)

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Figure 7b. Minimum Lateral Clearance in relation with Velocity (Cars)

Figure 7c. Average Lateral Clearance in relation with Velocity (Cars)
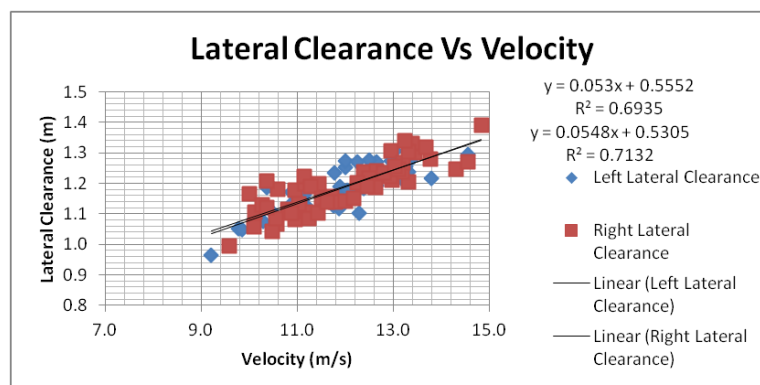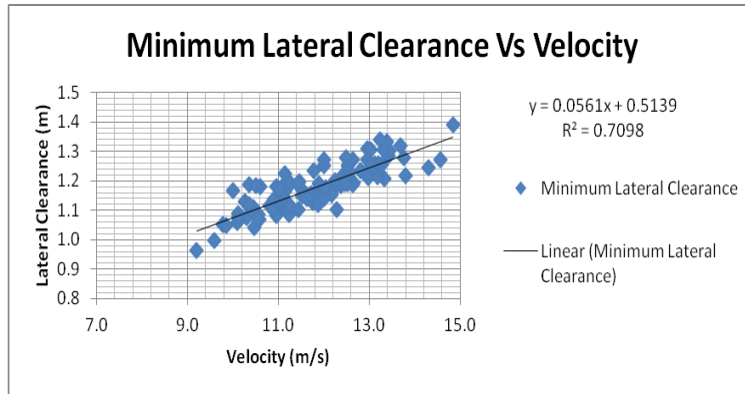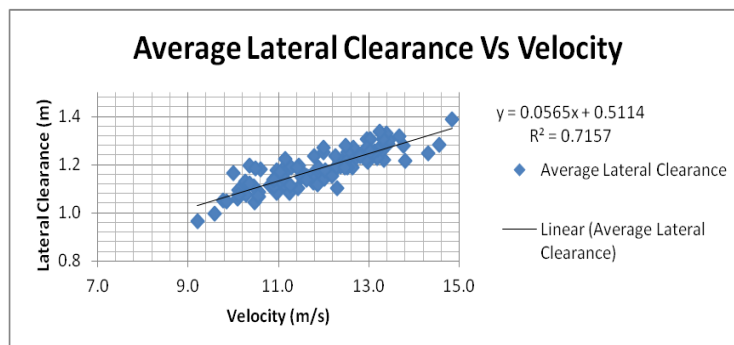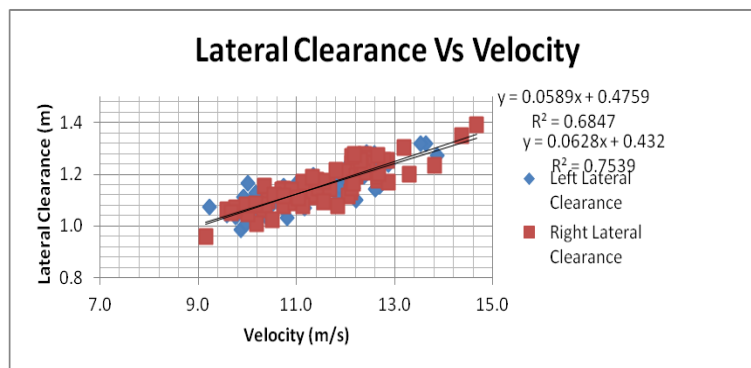
4.1.1.2 SUVs and Vans

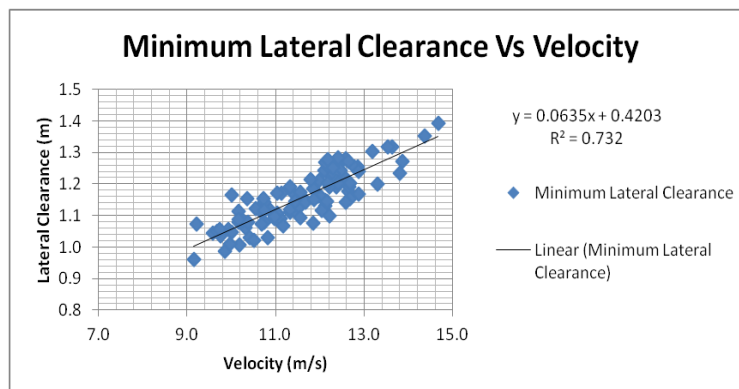Figure 8a. Lateral Clearance in relation with Velocity (SUVs & Vans)

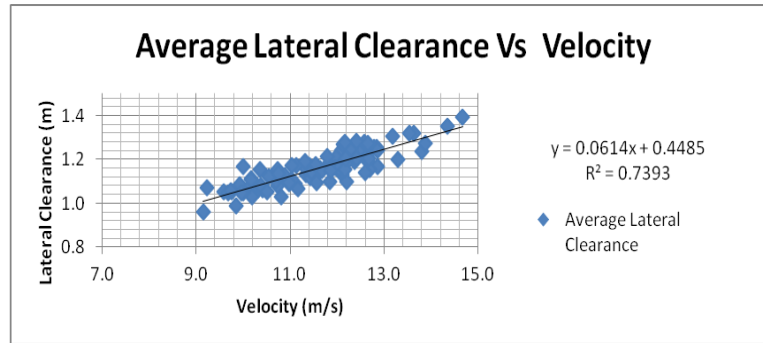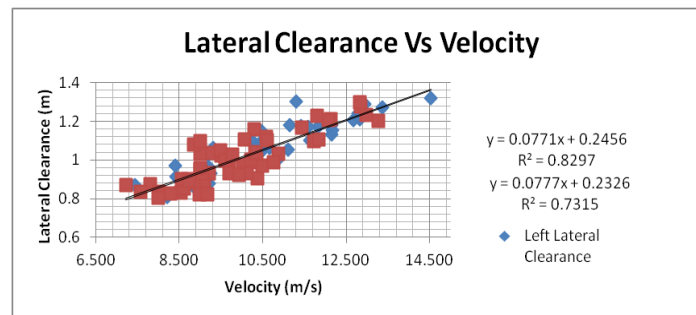Figure 8b. Minimum Lateral Clearance in relation with Velocity (SUVs & Vans)

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Figure 8c. Average Lateral Clearance in relation with Velocity (SUVs & Vans)

4.1.1.3 Buses and Trucks

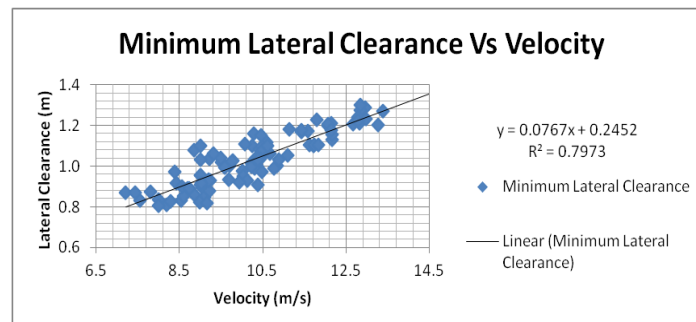Figure 9a. Lateral Clearance in relation with Velocity (Buses & Trucks)

Figure 9b. Minimum Lateral Clearance in relation with Velocity (Buses & Trucks)
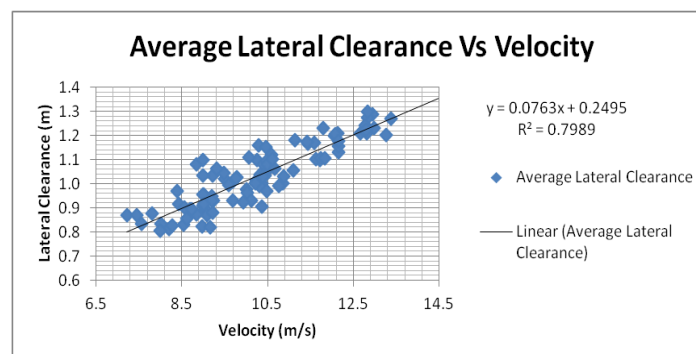

Figure 9c. Average Lateral Clearance in relation with Velocity (Buses & Trucks)

4.1.2. Frequency Distribution of Lateral Clearance

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Figure 10 shows the frequency distribution of lateral clearances wherein lane changes were made by vehicles, regardless of type. The frequency distributions by vehicle type are presented in Figures 11 to12. It can be observed from the Figures that buses and trucks generally maintain smaller lateral clearances during lane changes.
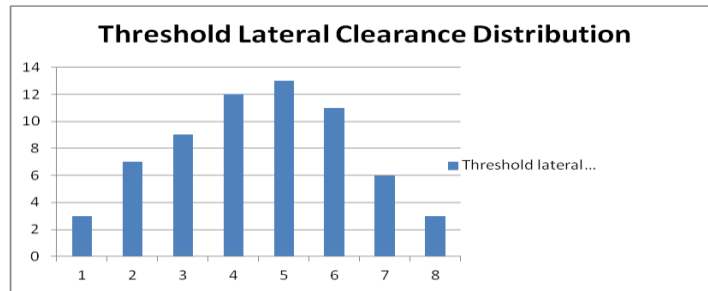

Figure 10. Threshold Lateral Clearance Distribution (all vehicles)
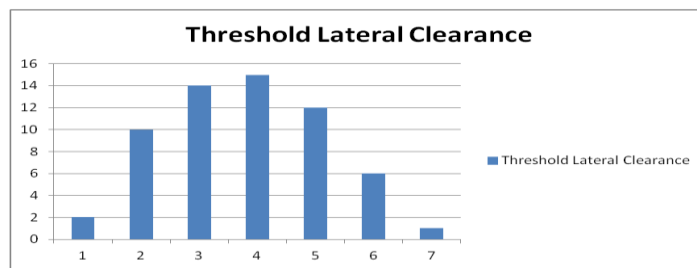

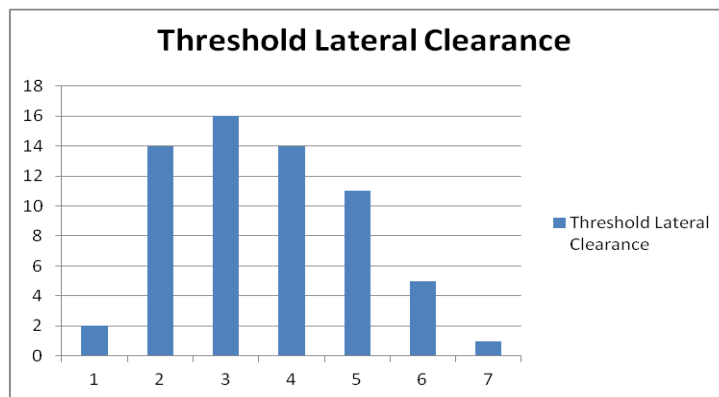Figure 11. Threshold Lateral Clearance Distribution for Cars

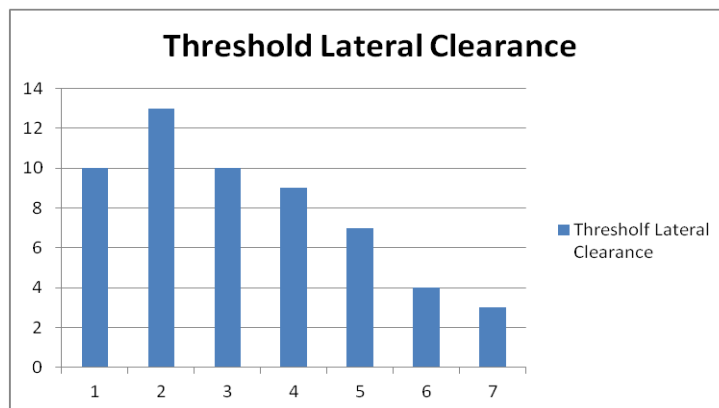Figure 12. Threshold Lateral Clearance Distribution for SUVs and Vans

Figure 13. Threshold Lateral Clearance Distribution for Buses and Trucks

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4.2 Threshold Velocity Difference

The velocity difference of the subject vehicle and the lead vehicle is a critical element on the formulation of the lane changing model being used in the local simulation which is the Minimizing Overall Braking Induced by Lane Changes (MOBIL). Before a vehicle decides to transfer lanes, one of the preliminary factors that trigger the said circumstance is the velocity of the lead vehicle. If the speed of the lead vehicle is too slow for the subject vehicle, a lane transfer might occur. On another hand, if the speed of the lead is fast enough in comparison with the velocities of the adjacent lanes, then the subject vehicle will remain in its initial lane. In relation with this, there must have a critical velocity difference between the subject vehicle and lead vehicle that if exceeded will instigate a lane change. The values below were the results of the field surveying executed along EDSA.

Mean relative velocity for lane changing	0.853 m/s
Variance / Std. Deviation	0.377 / 0.614

4.2.1 Frequency Distribution of Velocity Difference

Sturges was again used for the calculation of the number of class intervals. The x axis represents the threshold relative velocity while y axis represents the number of vehicles. Class 1 has intervals of small limit velocity difference. The value of the discussed parameter increases from Class 1 to Class 7. The frequency distribution generated in Figure 14 suggests that majority of the lane changes occurs when the difference in velocities between the lead and subject vehicle are small. This generalization is completely plausible since there is greater possibility of lane changing if the speeds of the interacting vehicle are almost of the same speed.
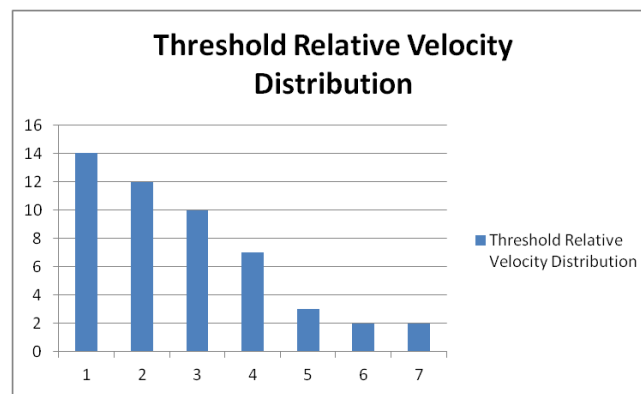
Figure 14. Threshold Velocity Difference

4. 3 Comparisons of Parameters

The measured lateral clearance along EDSA was compared to the lateral in other countries. The results obtained by Gunay on his study of Car Following with Lateral Discomfort will be used for the analysis. From the perspective of his research, the lateral clearance available limits the speed of the vehicles. For this reason, to compare the two results, the independent variable would be the lateral clearance while the dependent variable would be the speed of the vehicles. The data gathered by Gunay were taken at Germany and Britain. From the three plots, Britain have the highest slope with 55-59, followed by Philippines with a slope of 40.84, and then followed by Germany with the lowest slope that ranges from 20-24. The results obtained in Gunay's research are similar with the data acquired in EDSA in terms of the correlation of the lateral clearance and speed of the vehicle which is linear. However, the plots also suggests that in Germany , drivers are more cautious since they maintain a larger lateral clearance at a given speed as implied by its

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smaller slope compared to the Britain and Philippines. Using the slope as basis of risk factor, Britain has the highest risk in terms of lateral collision.
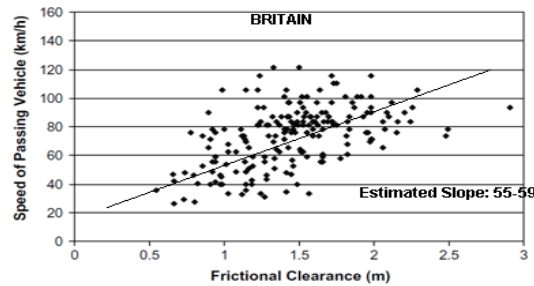

Figure 15. Speed of Vehicle in relation with Frictional Clearance (Britain)
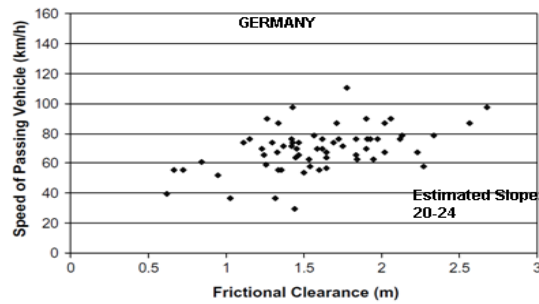(Source: Car Following with Lateral Discomfort – Gunay)

Figure 16. Speed of Vehicle in relation with Frictional Clearance (Germany)
(Source: Car Following with Lateral Discomfort - Gunay)
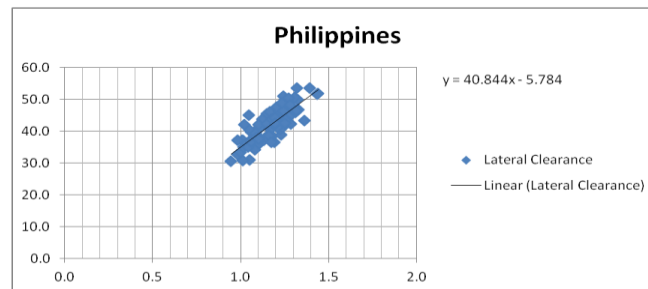

Figure 17. Speed of Vehicle in relation with Frictional Clearance (Philippines)

The parameters acquired in EDSA were also compared in the lateral clearance in India. Figure shows the plots of the transverse clearance with respect to a certain speed of the passing vehicle. The estimated slope for the lateral clearance in India is 0.01 to 0.014 which is too small compared to the slope of the parameter in the Philippines which is 0.0595. In terms of risk factor, drivers in India tend be less careful since they sustain smaller lateral clearance at given speed. The outcome is plausible since drivers in India are space-oriented instead of lane oriented. As long as there are spaces available, they will to insert their vehicles thus increasing the risk for lateral collision.

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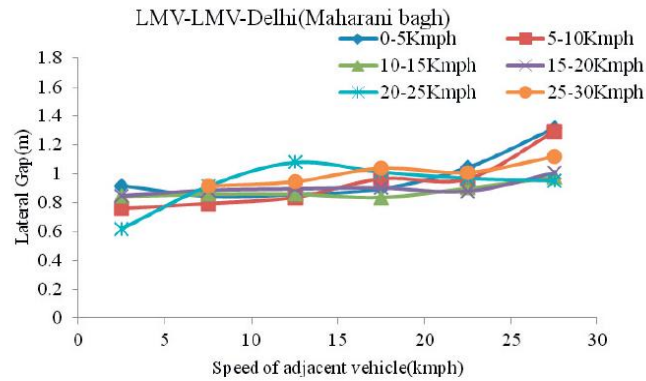Quezon City, Philippines, 8 August 2016

Figure 18. Lateral Gap vs Speed of Adjacent Vehicle

(Source: Analysis of Lateral Gap Mantaining Behaviorof Vehicles in HeterogeneousTaffic Stream)

7. Conclusion

The value of the threshold lateral clearance maintained by vehicles is not a discrete value. Instead, the lateral clearance depends upon the speed of the subject vehicle. The two variables have a linear relationship. As the velocity of the subject vehicle increased, the required lateral clearance also increased.

For the threshold velocity difference, it was found that the smaller the discrepancies in velocities between the lead vehicle and the subject vehicle, the greater its potential to transfer lanes. The threshold mean velocity difference that tends Filipino drivers to change lanes is 0.853 m/s.

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TSSP
2016

23rd Annual Conference of the
Transportation Science Society of the Philippines
Quezon City, Philippines, 8 August 2016

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