

CALIBRATION AND VALIDATION OF A WEATHER-RESPONSIVE ESTIMATION AND PREDICTION SYSTEM OF A MACROSCOPIC TRAFFIC FLOW MODEL

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Abstract: This study evaluates the METANET model's capability to simulate traffic dynamics under normal and adverse weather conditions, confirming its reliability in forecasting real-world traffic behavior, particularly in regions with frequent rainfall. Calibration and validation results demonstrate METANET's adaptability, with critical parameters such as free-flow speed, capacity, and critical density significantly influencing performance. Adjusting these parameters enhances METANET's responsiveness to weather-induced traffic flow variations, providing a robust foundation for developing weather-responsive traffic management systems. The study successfully replicates METANET's predictive performance using traffic data from Bangkok, Thailand, marking the first application of the model in this region. Findings suggest that accounting for traffic flow heterogeneity only marginally improves accuracy, likely due to the limited study area and its relatively homogeneous traffic conditions. Validation results indicate that weather-specific modeling outperforms general models, effectively capturing congestion onset and dissipation while accurately predicting spatiotemporal traffic variations. Weather-adaptive METANET models demonstrate improved accuracy in tracking congestion waves and replicating flow-density relationships under varying rainfall intensities. The research underscores the model's sensitivity to key traffic parameters, emphasizing the necessity of incorporating weather considerations into traffic flow modeling for more precise predictions.

1. INTRODUCTION

Weather conditions impact various road conditions, including traffic demand, safety, operations, and flow. This paper proposes including rainfall factors in macroscopic traffic flow modeling, calibrated and validated using field data. This model aims to calculate the effects of rainfall on traffic flow dynamics, which has potential applications for traffic control and operations.

Macroscopic traffic flow theory is used to predict states of traffic at any segment of a highway based on initial conditions. The model treats the flow of traffic as a fluid rather than focusing on individual vehicles, using three main variables - average speed,

density, and flow - to describe traffic stream characteristics. Macroscopic traffic flow models are classified as first-order, second-order, or higher-order models, depending on the number of differentials. In literature, two widely used macroscopic models are used. The first is the CTM or the Cell Transmission Model, and the other is the METANET model. In a systematic review of well-known papers on macroscopic traffic flow modeling by Wang et. al (Wang et al., 2022), they looked into 32 papers, of which 80 % either used CTM or METANET.

The Lighthill–Whitham–Richards (LWR) model (Lighthill & Whitman, 1955; Richards, 1956) is a popular first-order model, employing a single PDE in expressing the flow following the law of conservation of vehicles. In a well-known paper by Daganzo (Daganzo, 1994), he introduced the Cell Transmission Model (CTM), which represents the road as a series of discrete cells, each of which can hold a certain number of vehicles. Vehicles move between cells according to specific rules, allowing the model to capture traffic flow, congestion, and other traffic dynamics in a simplified yet effective manner. It is a discretized version of LWR, which can be solved analytically due to its piecewise linear Fundamental Diagram (FD). While both can determine the uncongested and congested side of the FD representing normal traffic and congestion, they cannot replicate complex phenomena. Examples of this are the capacity drop phenomenon and scattering in the q-k (flow density) diagram.. Higher-order models have solved these limitations. Several approaches have been developed to enhance the LWR model and its ability to simulate complex traffic phenomena.

The Payne-Witham (PW) Model, introduced by Payne (Payne, 1971) and Whitman (Whitman, 1974), derives the equation for acceleration using the expanded Taylor series of a car-following model. It offers insights into complex traffic phenomena such as scattering in the flow-density diagram. A discretized version of this model, METANET, was proposed by Messmer and Papageorgiou (Messmer & Papageorgiou, 1990) and is widely used for large-scale network applications and control purposes (Kontorinaki et al., 2017; Kotsialos et al., 2002; Papageorgiou et al., 2010; Spiliopoulou et al., 2014, 2017). The METANET model is a macroscopic discrete second-order model that was firstly applied to the Boulevard Périphérique in Paris. The name METANET, acronym for “Mod’ele d’Ecoulement de Trafic sur Autoroute NETWORKs” in French which translates to "Traffic Flow Model on Highway Networks", was firstly associated with the simulation tool for the freeway network, but it is now adopted to generically indicate the second-order traffic flow model.

In the study of Spiliopoulou et al. (2014), which compared traffic flow modeling using the Cell Transmission Model and METANET, they showed that the second-order model METANET performed better than CTM. Kontorinaki et al. (2017) also conclude that METANET outperforms other macroscopic models such as CTM and LWR and their extensions. In a comprehensive benchmarking of macroscopic traffic flow models conducted by Mohammadian et al. (Mohammadian et al., 2021), they compared the performance of the following models: LWR, LWR with extensions, CTM, CTM with extensions, METANET, Gas-Kinetic Theory, and Generic Second-Order Modeling. The models were assessed for their effectiveness in tracking congestion. Their findings indicated that METANET demonstrated the best performance overall, particularly excelling in modeling ramp-merging congestion.

90 This paper focuses on the METANET model, a discretized and improved variation of
91 the LWR model combining the characteristics of the PW model. METANET is chosen
92 for its three distinct dynamic functions that predict key traffic flow parameters: flow,
93 speed, and density. Its form and convenient discretization intervals allow to facilitate
94 the integration of field-collected data. The METANET model's clear analytical
95 properties, including a specific space-time form with improved differentiable functions,
96 make it suitable for real-time freeway traffic control operations. METANET has been
97 applied across a range of areas, including Freeway Traffic Flow Modeling, where it has
98 been utilized to model freeway traffic dynamics (Kontorinaki et al., 2017;
99 Mohammadian et al., 2021; Spiliopoulou et al., 2017; Wang et al., 2022), variable speed
100 limit control which was applied to manage and control variable speed limits (Wang et
101 al., 2021), ramp metering where the model has been used to optimize strategies for
102 metering of ramps (Kan et al., 2016; Wang et al., 2014, 2021), and traffic state
103 estimation and prediction which has been employed for estimating and predicting traffic
104 states (Wang et al., 2022; Zhao, 2021).

105 The weather-specific METANET model is calibrated and integrated into a validation
106 model using field data. This paper makes four main contributions: First, it considers
107 different weather conditions for calibration and applies them to various macroscopic
108 traffic variables during validation, whereas, to our knowledge, previous studies have
109 typically focused only on the applicability of weather-specific parameters and not on
110 comparing them with other weather conditions. Second, it utilizes real field data rather
111 than simulation data to provide clear quantitative results, demonstrating the significant
112 impact of weather on freeway traffic dynamics. Lastly, it demonstrates the effects of
113 considering multiple fundamental diagrams for every section of the highway.

114 2. METHODOLOGY

115
116 This paper focuses on the METANET model, a widely used macroscopic traffic flow
117 model shown to outperform other models like the Cell Transmission Model (CTM) in
118 accurately tracking freeway congestion. METANET is based on a discretized, enhanced
119 version of the Payne-Witham model. It predicts key traffic parameters (flow, speed, and
120 density) through three dynamic functions, making it suitable for real-time freeway
121 traffic control. This paper aims to calibrate and validate METANET using a section of
122 Bangkok's Burapha Whiti Expressway, which includes ramps and is subject to
123 recurring congestion.

124
125 METANET divides the highway into sections and computes traffic states at each
126 discrete time step. It uses equations involving density, flow, and speed, enhanced by
127 additional terms for accurate on-ramp merging and lane-drop modeling. Calibration
128 aligns model predictions with actual traffic data, while validation tests model
129 performance on an independent dataset, focusing on average speed and flow as
130 performance metrics. This process is optimized using the Nelder-Mead algorithm,
131 selected for its efficiency in achieving accurate parameter estimates.

132
133 Traffic data, including flow, speed, and density, were collected every five minutes
134 throughout 2022, covering both dry and rainy conditions. Two days of data were used
135 for calibration (one with rain, one without), and six additional days were used for
136 validation, ensuring no incidents or sensor failures during the study period.

137
 138 METANET employs the discretization and enhancement of the form of the **Payne**
 139 model. It treats the highway section as continuous numbered sections i (section i is
 140 downstream of section $i-1$), each with specific lengths L_i , and lane numbers λ_i . Time
 141 is subdivided into equal gaps of duration T . In every discrete time step, time $k = 0, 1, \dots,$
 142 K , METANET computes the density, flow, and speed for each section i using the
 143 equations:
 144

$$145 \quad \rho_i(k+1) = \rho_i(k) + \frac{T}{L_i \lambda_i} [q_{i-1}(k) - q_i(k)] \quad (1)$$

$$146 \quad q_i(k) = v_i(k) \rho_i(k) \lambda_i \quad (2)$$

$$147 \quad v_{i+1}(k) = v_i(k) + \frac{T}{L_i} v_i(k) [v_{i-1}(k) - v_i(k)] + \frac{T}{\tau} [V^e(\rho_i(k)) - v_i(k)] - \frac{v T [\rho_{i+1}(k) - \rho_i(k)]}{\tau L_i [\rho_i(k) + \kappa]} \quad (3)$$

148

149 Where the model parameters are a time constant (τ), an anticipation constant (v), and a
 150 mode parameter (κ). $V^e(\rho_i(k))$ represents the fundamental diagram and is determined
 151 using the following equation:

$$152 \quad V^e(\rho_i(k)) = v_{f,i} \exp \left[-\frac{1}{a_i} \left(\frac{\rho_i(k)}{\rho_{cr,i}} \right)^{a_i} \right] \quad (4)$$

153 where $v_{f,i}$ represents the free-flow speed, $\rho_{cr,i}$ represents the critical density
 154 corresponding to the maximum flow condition, and a_i another model parameter. It
 155 must be noted that the average speed calculated must be at least v_{\min} .

156 For more accurate merging and lane-drop modeling, enhancements were proposed by
 157 Papageorgiou et al. (1989). Specifically, the model incorporates two additional terms
 158 to capture these effects better. One notable term addresses the on-ramp merging impact
 159 by adjusting the traffic flow dynamics for the influence of merging cars. The following
 160 term is then added

$$161 \quad -\delta T q_\mu(k) v_i(k) / L_i \lambda_i (\rho_i(k) + \kappa)$$

162 into eq. (3) for the on-ramp segment. A further parameter δ is included, and q_μ
 163 represents the number of vehicles entering the ramp. To include the consideration of
 164 the effects of lane-changing, the term

$$165 \quad -\phi T \Delta \lambda \rho_i(k) v_i(k)^2 / L_i \lambda_i \rho_{cr,i}(k)$$

166 is included to eq. (3) for the consideration of the immediate section upstream when
 167 there is a dropping of lanes, a model parameter ϕ is further added where $\Delta \lambda$ refers to
 168 the number of lanes that were dropped.

At the change of geometric characteristics, such as the presence of ramps, the flow is separated into various sections that exit on the highway based on a defined turning rate $\beta_j(k)$. Additionally, for sections entering the change in highway geometry, a density downstream $\rho_i(k+1)$ is required in eq. (3) to account for the influence of downstream traffic conditions. Given that bifurcations lead to two downstream sections, the following is used to determine the downstream density for section i at the bifurcation as proposed by Messmer & Papageorgiou (1990):

$$\rho_{i+1}(k) = \frac{\sum_{\mu \in O_i} \rho_{\mu}^2(k)}{\sum_{\mu \in O_i} \rho_{\mu}(k)} \quad (5)$$

where $\rho_{i+1}(k)$ represents the computed downstream density under consideration utilized in eq. (3) for section i . Meanwhile, $\rho_{\mu}(k)$ denotes the section density downstream of considered segment, where O_i is the set of sections which exits the highway. The quadratic average used in eq. (5) considers the potential congestion spillback to the section. The eq. (5) does not require any additional calibration parameters. For this study, the actual densities on off-ramps are given as boundary conditions to the model. Consequently, any increase in the density of the off-ramp directly affects the average speed of the upstream mainstream area through equations eq. (3) and eq. (5). For merging locations with on-ramps, the model incorporates the actual on-ramp flows as direct input, treating on-ramps as integrated parts of the sections rather than modeling them as separate entities.

METANET modeling includes various parameters whose values can vary depending on factors like the network geometry, driver behavior, truck percentage, and weather conditions at different freeway sites. Therefore, the accuracy and reliability of these models depend on correctly specifying these parameter values. Calibration of the models is often necessary to ensure they are suitable for real-world applications while validation tests the accuracy of the model.

2.1 Calibration and validation of models

Before applying traffic models to practical applications like traffic monitoring and management, they must undergo calibration and validation using actual traffic data. Calibration adjusts model parameters to minimize the difference between predictions and observed data, ensuring the model accurately reflects current traffic conditions. Validation tests the calibrated model's predictive performance using an independent dataset, assessing its ability to predict future traffic. Both processes are critical: calibration without validation has limited value, as it only fits the model to a specific dataset without ensuring broader applicability.

Calibration also estimates parameters not directly observable in the dataset, especially in higher-order models like METANET, which include empirical terms to enhance modeling capabilities. This process typically involves solving nonlinear systems through optimization techniques to minimize the difference between model outputs and actual data using a cost function. Parameter values are selected from an admissible range defined by prior experience and physical meaning.

Validation uses a different dataset (e.g., from another day) within the same study area to compare model outputs with actual data, ensuring the model performs reliably across

various traffic conditions and time periods. While flows are easier to model due to conservation equations, accurately predicting average speeds across highway segments remains challenging.

Calibrating a least-squares optimization problem nonlinearly often has multiple local minima as illustrated by Ngoduy & Maher (2012) in the calibration of second-order traffic models using the continuous cross-entropy method, making gradient-based methods unsuitable. Spiliopoulou et al. (2017) demonstrated that different optimization techniques can arrive at solutions to the estimation problem for METANET. They have employed three commonly used optimization algorithms. The first is the Nelder-Mead (N-M) algorithm (Lagarias et al., 1998; Nelder & Mead, 1965) which is deterministic. The next one is the stochastic Genetic Algorithm (GA) (Whitley, 1994). Lastly, they also considered the cross-entropy method (Rubinstein & Kroese, 2004). All three algorithms were able to converge to a solution set. Nevertheless, their results show that the Nelder-Mead algorithm performs 257 times faster than the genetic algorithm and 242 times faster than the cross-entropy method. This is an important finding since the computation time must be considered in traffic flow modeling, especially because it often involves real-time applications. Therefore, the Nelder-Mead Algorithm is used in this study for the determination of the parameters. The algorithm is also well-suited for finding acceptable local minima, or potentially even the global minima, in complex, multi-dimensional optimization landscapes.

2.2 The Nelder-Mead algorithm

The Nelder-Mead algorithm (Nelder & Mead, 1965) is a renowned algorithm for the optimization of multidimensional systems with unconstrained conditions. For this study, we have particularly adapted a version for constrained optimization as described by Spiliopoulou et al. (2014). This method is advantageous because it requires no derivative information, making it applicable for problems with nonlinear and discontinuous cost functions.

The algorithm operates using a simplex with n number of dimensions and $n+1$ number of vertices. Each vertex represents a potential solution and has an associated cost function value. The Nelder-Mead method begins with an initial simplex and iteratively transforms it to increase the predictive accuracy at the vertices. In every iteration, the method will sort the solutions by their cost function values, calculate the centroid, and then update the solution by reflecting, expanding, or contracting the worst vertex. If these transformations do not yield improvements, the algorithm performs a shrinkage towards the best vertex, generating new vertices.

The algorithm's performance is influenced by the four parameters of σ for shrinkage, χ for expansion, ξ for reflection, and Υ for contraction. The recommended values for these parameters are 0.5, 2, 1, and 0.5, respectively (Spiliopoulou et al., 2014). The algorithm may sometimes perform many iterations without significant improvements. To address this, multiple restarts with a limited number of iterations can be used as a heuristic solution. This technique was employed in this paper by doing 5 runs for each calibration and comparing the respective performances.

2.3 Measurement of performance

In this context, measuring performance involves assessing key variables used to evaluate the results of model calibration. For macroscopic traffic flow modeling, the primary variables are flow in veh/hr, density in veh/km, and average speed in km/hr. The calibration process is compared against the actual dataset, which is also represented by these traffic flow variables. Therefore, evaluating calibration results based on these variables is natural. From the modeling results of other papers, it is advisable to focus on average speeds during the calibration procedure. This is because empirical observations suggest predicting flows is relatively straightforward due to the conservation law, even if speed matching is not perfect.

Second, it has been found that if the modeled speeds match the actual speeds in the segment considered adequately, the flow predictions are generally satisfactory. Thus, focusing on speed matching is crucial and often more challenging. Lastly, measuring densities directly or modeling occupancies is difficult, with data often being incomplete or inaccurate.

2.4 Test Network, Evaluation, and Traffic Data

A highway network is graphically represented where links correspond to segments of the highway with consistent characteristics, such as uniform lane count, grade, and curvature. Nodes are used to indicate significant changes in the roadway's geometry including ramps and lane reductions. If a segment exhibits varying characteristics, it is divided into multiple links, each separated by a node.

For computational modeling, the time horizon is divided into 5-second intervals. To ensure the stability of the numerical method, the length of each segment and the time interval must satisfy the celebrated Courant-Friedrichs-Lewy (CFL) condition (Courant et al., 1928; de Moura & Kubrusly, 2013; Sanz-Serna & Spijker, 1986).

The study area considered for this paper is a segment of the Burapha Whiti Expressway in Bangkok, Thailand (station 12+400 to 19+500). This highway stretch includes on-ramps and off-ramps. This part of the expressway was considered for the calibration and validation of the traffic flow model because of the presence of the ramps. This is also the segment of the highway nearest to the automatic weather station. Only this span of the highway was considered to make the effects of weather more pronounced as different parts of the highway far from the weather station may experience a different effect on the traffic flow in consideration of weather. To model the study area by use of the METANET model, the highway is represented with four nodes and three links. Each node represents an area with a change in geometric characteristics in the highway. The homogenous highway segments in between are denoted by links. Figure 1 displays the length, number of lanes, ramp locations, and the location of the microwave radar detectors represented by bullets.

The dataset for this study was gathered at the Expressway Authority of Thailand, which operates country's expressway system. It includes flow, speed, and density for each microwave radar station gathered every 5 minutes. It is observed that recurring congestion is evident in this area in the morning rush hours due to the presence of ramps. 2 days were selected for the calibration process, representing each for good and

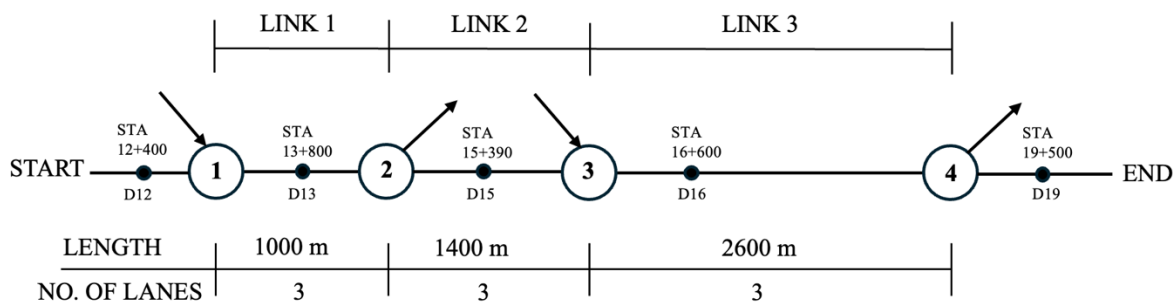
bad weather conditions representing the occurrence of rainfall during the gathering of data while 6 days were considered for validation (3 for each weather condition). It is important to note that the primary criterion for selecting these 8 days was that no incidents or detector failures occurred during the morning hours of 5–12 AM on the examined freeway stretch, conditions which could not be replicated by any traffic flow model. Figure 2.1 illustrates the detailed layout of this expressway and the locations of traffic sensors, represented by black dots. Data for analysis were collected in 2022, including days with and without precipitation.

323
324

325 **Figure 1**

326

327 *Representation of the Study Area Considered in Burapha Whiti Expressway, Bangkok,*
328 *Thailand.*



329

330 2.5 Novel Considerations

331 This paper offers a comprehensive examination of macroscopic traffic flow modeling,
332 focusing on both calibration and validation. Unlike previous studies, this research
333 provides a more detailed analysis of model calibration, addressing critical aspects such
334 as congestion tracking, capacity reduction, and the impact of weather conditions on
335 prediction accuracy. It also includes extensive validation results from the same case
336 studies, demonstrating that the METANET model accurately captures traffic flow
337 dynamics, particularly concerning weather-specific conditions.

338 Despite the significance of calibrating models with real traffic data for accurate
339 application, there is a scarcity of research dedicated to model calibration and validation
340 (Wang et al., 2022). Furthermore, the numerical computation of these models
341 necessitates space-time discretization, making simplified and analytically tractable
342 models highly beneficial for practical applications.

343 This paper aims to address four key issues in traffic flow modeling:

- 344 a. **Traffic Flow Inhomogeneity:** stretches of a highway usually exhibit inhomogeneity
345 in traffic flow due to variations in key traffic flow parameters caused by several
346 factors. To effectively model such variations, a model must consider different
347 fundamental diagrams to represent a section of the highway with the same traffic
348 and geometric characteristics.

- 349 b. Congestion Tracking: this refers to a model's ability to reproduce and predict when
350 recurrent congestions emerge and propagate. A model should accurately reproduce
351 the dynamics of congested traffic flow across spatial and temporal scales. The
352 effectiveness of a model in reflecting real-world traffic is determined by how its
353 structure is represented mathematically and by the parameters the model includes.
354 When it is established, the focus shifts to calibrating parameters to ensure the model
355 can accurately describe the entire evolution of traffic conditions.
- 356 c. Effects of Weather on Macroscopic Traffic Flow Modeling: the performance of
357 models that undergo calibrations under an unspecified weather event may not
358 accurately represent the dynamics of traffic under different weather conditions in
359 the same location, as key traffic parameters can be significantly affected. In the
360 literature, only the work of Bie et al. (2017) specifically addressed and investigated
361 the impact of weather parameters on traffic dynamics using METANET by
362 introducing a weather factor into macro traffic state prediction. There is limited
363 quantitative evidence on the broader impact of weather on model performance. This
364 research will also address model parameter transferability, evaluating whether a
365 calibrated model remains applicable to new datasets from different times or weather
366 conditions, a topic that has not been fully explored in the literature. The focus will
367 be on determining if a model developed with normal weather data can be effectively
368 used for rainy or other adverse weather conditions, and vice versa.

369 This research seeks to provide empirical evidence and enhance the understanding
370 of these issues, which are currently supported more by qualitative observations than
371 by quantitative analysis.

372 **3. RESULTS AND DISCUSSION**

373 **3.1 Model Calibration Result under Normal Weather Conditions**

374 The different results of the calibrations are detailed in this section, focusing first under
375 normal conditions. The data for June 9, 2022, was used for this. For not accounting for
376 traffic flow heterogeneity, the specific results are illustrated in Figures 3.1 and 3.2.
377 When considering for heterogeneity, the summary of the performance with specific
378 results illustrated in Figures 3.3 through 3.4.

379 The Nelder–Mead algorithm was employed for calibration with the following settings:
380 $\xi = 1$, $\chi = 2$, $\gamma = 0.5$, and $\sigma = 0.5$ (for further details, see Section 2.2). The algorithm
381 was terminated based on either the convergence of the cost function or the convergence
382 of the acceptable simplex, both when the tolerance level reaches 0.1, and after 500
383 iterations.

384 **3.1.1 Not Considering Heterogeneity**

385 As discussed in the previous sections, not accounting for traffic flow inhomogeneity
386 means that only 1 fundamental diagram (FD) is considered for the whole study area.
387 Analysis of the measurement data revealed that the highway under consideration has
388 the same geometric characteristics.

389 Five calibration runs were conducted, and their performance is measured in terms of
390 Mean Absolute Percentage Error (MAPE) for all the detectors. For each calibration run,
391 the algorithm begins with specified initial values and randomly generates the next

values based on these initial values. As described in the previous sections, the working simplex consists of $n + 1$ vertices, where n represents the number of parameters being calibrated. Using a physically reasonable initial vertex is preferable to expedite algorithm convergence. The results show that the best performance of all the calibration runs has a MAPE of 1.3943 %, 3.1043 %, and 5.7205 % for Detectors 13, 15, and 16, respectively. Nevertheless, the difference in the performance index between each run is not very significant, showing that the algorithm converges to almost the same optimal parameters. The best run will be used for further analysis.

Table 3.1 presents the optimum parameter values calculated by the Nelder-Mead algorithm for the best calibration run. τ is the relaxation time parameter in second (s) which impacts how fast the average speed can cope with the speed in equilibrium that is computed in the FD, ν is an anticipation parameter in km^2/h controlling the backward movement of the congestion wave, δ is a parameter controlling the merging mechanism in h/km , Φ is the parameter responsible for the dropping of lanes in h/km , κ is an additional model parameter in $\text{veh}/\text{km}/\text{lane}$, ν_{\min} is the minimum value of the speed in km/hr , ν_f is the free-flow speed in km/hr , ρ_{cr} refers to the critical density in veh/km , and q_{cap} means the capacity in veh/hr .

Table 3.1

Optimal Parameter Values for June 9, 2022 (Normal Weather Condition)

Model Parameters	Value
τ (s)	8.121
ν (km^2/h)	29.000
δ (h/km)	0.118
Φ (h/km)	0.00021
κ ($\text{veh}/\text{km}/\text{lane}$)	2.337
ν_{\min} (km/hr)	12.159
ν_f (km/hr)	83.225
ρ_{cr} (veh/km)	21.402
q_{cap} (veh/hr)	1781.210

Figure 3.1 shows the congestion tracking performance on June 9 using only 1 Fundamental Diagram. It illustrates the modeling outcomes for traffic flows and mean speeds. It presents the calibration results for flows and speeds at all sensor locations in the study area. In these figures, black refers to actual measurements, while red refers to the modeling results. The calibrated flow models closely predicted the actual data at different locations, while the mean speed models effectively matched when the congestion was formed and when it dissipated. It is evident that flow prediction is more accurate than with mean speeds. This is due to the governing conservation equation for traffic volumes which is unaffected by free flow speeds. On the other hand, average speeds are greatly affected by these.

It highlights the congestion events in the study area. The event started at detector 16 and spread downstream to detector 13 at around 6:45 AM to 8:30 AM. The congestion

event was accurately predicted in terms of spatiotemporal coverage. The results are considered satisfactory to be applied further. Note that the model has been arranged in downstream-to-upstream form.

Figure 3.2 shows the space-time maps depicting the coverage of the dynamics of flows and average speeds in the highway section, in which the y-axis represents the spacing requirements in the traffic flow direction. When comparing it to the actual data, the models under calibration effectively captured traffic flow dynamics and reproduced the emergence and dissipation of the congestion wave.

3.1.2 Considering Heterogeneity

Considering traffic flow inhomogeneity means that a unique fundamental diagram (FD) is assigned to a segment of the highway. This highlights how variations in key traffic flow parameters reflect traffic flow differences across the study area.

Five calibration runs were again conducted and their performance is measured in terms of MAPE for all the detectors. The same with the consideration of the same fundamental diagram for each section, the algorithm began with a similar initial value with the working simplex generated by randomly generating succeeding values around this initial point. The result shows best performance of all the calibration runs has a MAPE of 1.3152 %, 2.8912 %, and 5.5827 % for Detectors 13, 15, and 16, respectively. Again, the difference in the performance index between each run is not very significant which shows that the algorithm can converge to almost the same optimal values.

The modeling result in terms of congestion tracking is shown in Figure 3.3. It illustrates the modeling outcomes for traffic flows and mean speeds for June 9, 2022 considering heterogeneity. It presents the calibration results for flows and speeds at all sensor locations in the study area. In these figures, black refers to actual measurements, while red refers to the modeling results. The calibrated flow models in closely predicted the actual data at different locations, while the mean speed models effectively matched when the congestion was formed and when it dissipated. Flow prediction is more accurate than with mean speeds due to the conservation equation being the same as the result of considering only 1 fundamental diagram.

Figure 3.4 shows the space-time maps depicting the coverage of the dynamics of flows and average speeds in the highway section, in which the y-axis represents the spacing requirements in the traffic flow direction. When comparing it to the actual data, the models under calibration effectively captured traffic flow dynamics and reproduced traffic congestion with appropriate strength over the relevant spatiotemporal range. This highlights the congestion events in the study area. The event started at detector 16 and spread downstream to the detector 13 at around 6:45 AM to 8:30 AM. The congestion event was accurately predicted in terms of spatiotemporal coverage.

Figure 3.1 *Results of Model Calibration of flows and speeds at Different Sensor Locations – June 9,1 FD*

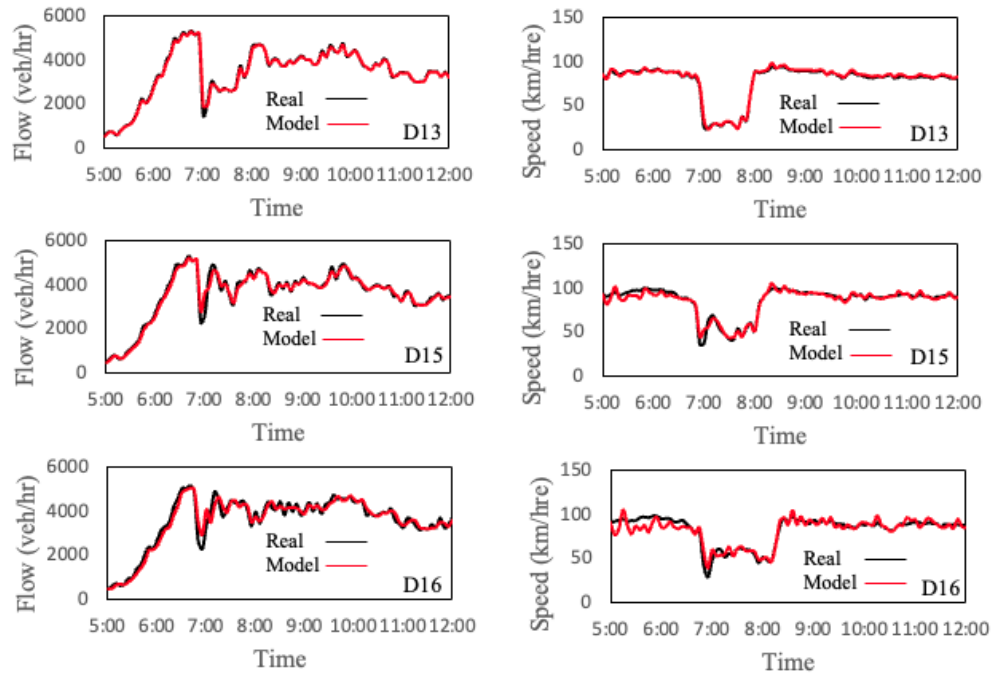


Figure 3.2 Space-time evolution of flows and speeds along the study area (a) real data; (b) using 1 Fundamental Diagram

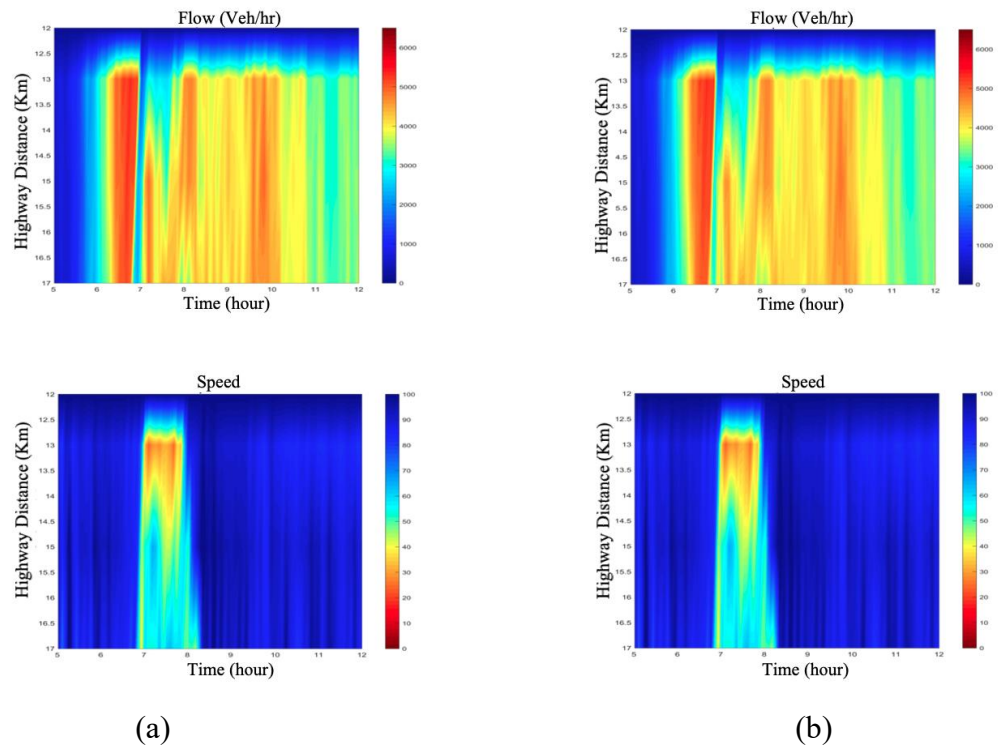


Figure 3.3 Results of Model Calibration Flows and Speeds at Different Sensor Locations – June 9, 3 FD

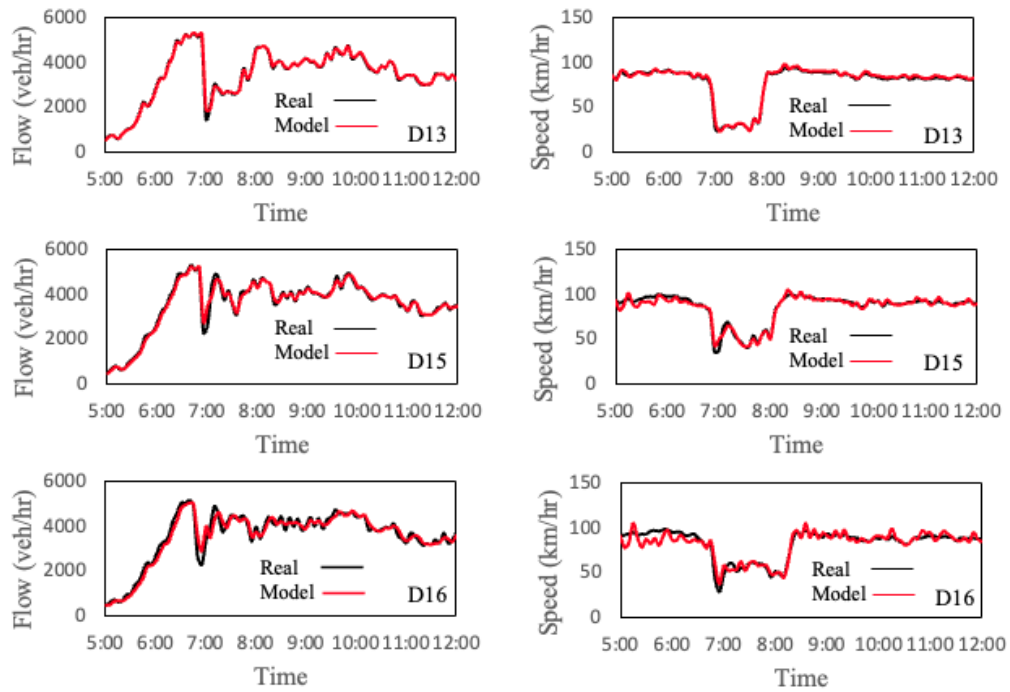
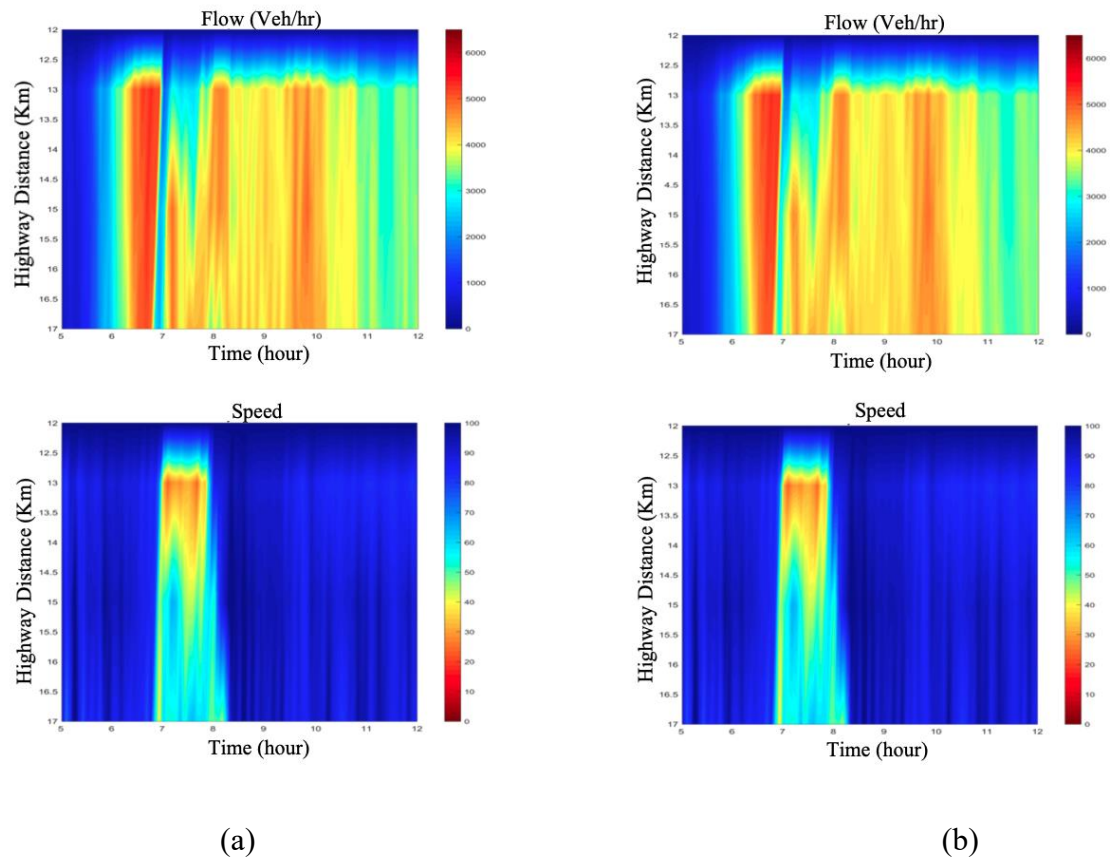


Figure 3.4 Space-time evolution of flows along the study area (a) actual data; (b) using 3 Fundamental Diagrams



3.2.2 Summary of results in consideration of traffic flow heterogeneity

Table 3.2 summarizes the performance of the best calibration runs in terms of MAPE of speed in km/hr for all the detectors in consideration of traffic flow heterogeneity. The result shows that considering traffic flow heterogeneity in traffic flow modeling has an advantage, as shown in the decrease in the value of MAPE. This result is expected because of the increase in the number of degrees of freedom. Nevertheless, this difference between the performances is only slight and can be considered not significant. This is different compared to the result obtained by (Wang et al., 2022) where the consideration of different fundamental diagrams resulted in a significant difference in the performance indicators. This can be explained by their use of the whole expressway network which consisted of different sections with different geometric characteristics. In the current study, the span of the highway considered consists of the same number of lanes and other geometric characteristics which justifies that they can have the same fundamental diagram. With this, further application in the next sections will only consider one fundamental diagram for the traffic flow modeling in the considered study area.

Table 3.2

Performance summary for 5 calibration runs on June 9, 2022 considering traffic flow heterogeneity

Heterogeneity Consideration	Detector No.	MAPE
1 FD	D13	1.3943
	D15	3.1043
	D16	5.7205
3 FD	D13	1.3152
	D15	2.8912
	D16	5.5827

The space-time diagram for flows and speed using 1 and 3 fundamental diagrams shows that both models were able to predict when congestion forms and dissipates in the spatiotemporal range and that the speed and flow prediction matches are both acceptable for further application. Overall, the congestion tracking results under normal weather were very satisfactory.

3.2 Model Calibrations under Rainy Weather Conditions

The calibration process was also performed using measurement data on a rainy day on Sept 26, 2022. The same 5 runs are completed and the calibration performance in terms of MAPE for speed is determined. The same as the calibration for normal weather conditions, there is not a significant difference in the MAPE showing that the algorithm converges to almost the same parameter values. The result shows that the performance of the calibration has a MAPE of 3.1673 %. The resulting parameters of the model are

shown in Table 3.4. It should be noted that there is a substantial decrease in the key traffic flow parameters of free-flow speed v_f , critical density ρ_{cr} , and q_{cap} . To illustrate, the free-flow speed, critical density, and capacity under normal weather conditions are 83.2 km/hr, 21.4 veh/km, and 1781.2 veh/hr, respectively. Under bad weather conditions, it was reduced to 72.3 km/hr, 21 veh/km, and 1510 veh/hr. respectively. This is expected since it has been proven in the literature that weather conditions affect the key traffic flow parameters and are consistent with empirical observations.

Table 3.4

Optimal Parameter Values for September 26, 2022 (Bad Weather Conditions)

Model Parameters	Value
τ (s)	6.25
ν (km ² /h)	27.99
δ (h/km)	0.20
Φ (h/km)	0.00
κ (veh/km/lane)	10.00
v_{min} (km/hr)	10.00
v_f (km/hr)	72.26
ρ_{cr} (veh/km)	21.00
q_{cap} (veh/hr)	1510.71

With the same study area which has homogeneous traffic flow, the modeling results exhibited similar calibration accuracy for flows on a rainy day compared to a non-rainy day. The model was still able to match the real flow and speed data including the tracking when congestion emerged and dissipated. However, the accuracy of speed calibration on rainy days was somewhat reduced.. The modeling results for flows and mean speeds are shown in Figure 3.5 while the space-time diagram for flows and speeds is illustrated in Figure 3.6.

3.3 Validation of METANET under Different Weather Conditions

Both the validation results for flows and speeds under different weather conditions are shown in this section. In each pair of the following validation processes, the optimal model parameters for each weather condition are used for a given date. It is first validated using calibration results from the same weather condition and another validation process is conducted using the calibration results from a different weather condition. Note that the accompanying parameters for a given weather condition are used (see Tables 3.1 and 3.3).

For each type of weather, 3 days were used for validation purposes. For normal weather conditions, June 17, 2022; June 20, 2022; and June 29, 2022 were used. For bad weather conditions, June 30, 2022; October 6, 2022; and October 7, 2022, were validated. For

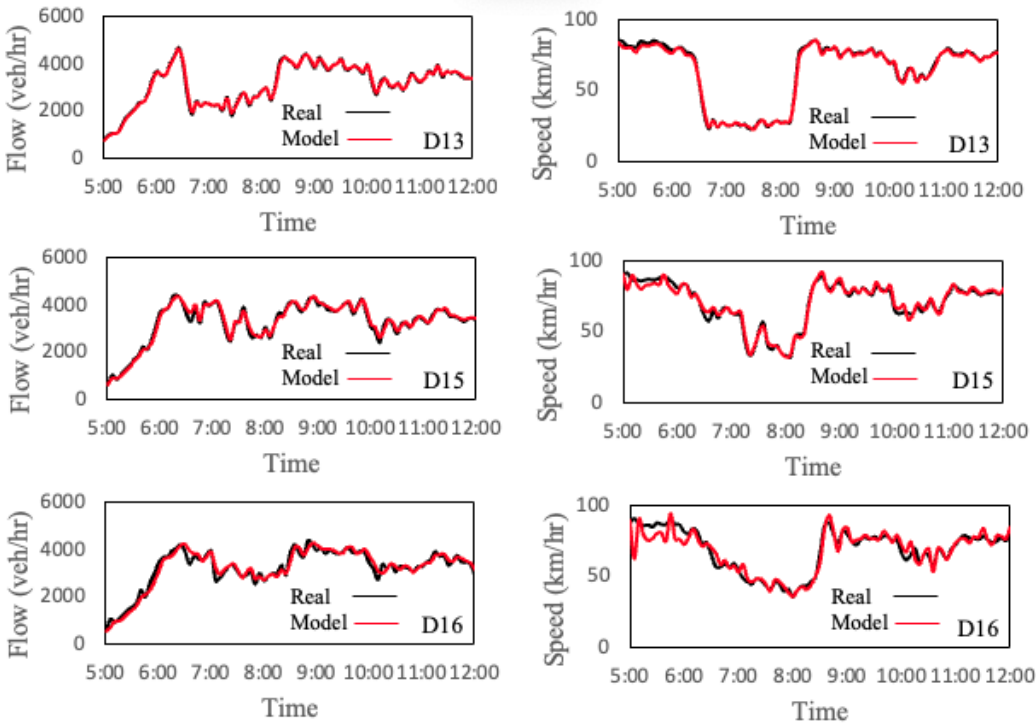
576 a given day, the modeling spanned from the same time interval during the calibration
577 process.

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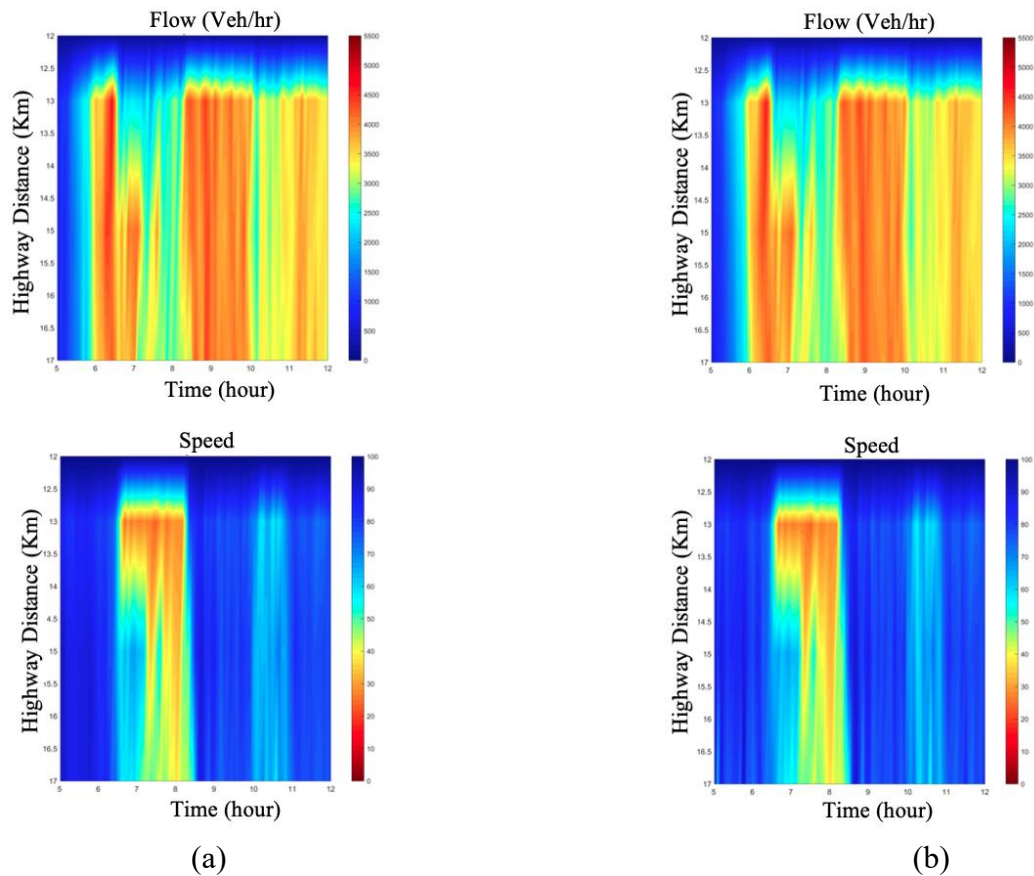
580 **Figure 3.5** Results of Model Calibration (Flows and Speeds) at Different Sensor
581 Locations – Sept 26

582



583

584 **Figure 3.6** Space-time evolution of flows along the study area on Sept 26 (a) real
585 data; (b) model



588 3.3.1 Validation on days with no rain

591 The validation of the model under normal weather conditions for 3 days on June 17,
592 2022; June 20, 2022; and June 29, 2022, is discussed in this section. Table 3.4 shows
593 the detailed quantitative error measurement in terms of MAPE for comparison.
594 Weather-specific means that the model parameters used in the validation are the
595 calibration results under normal weather conditions (see Table 3.1). On the other hand,
596 non-weather-specific means that the validation model parameters used are those
597 derived under the calibration of a day with bad weather conditions (see Table 3.4). It is
598 shown that weather-specific validation always performs better than the non-weather-
599 specific modeling results. To illustrate, the MAPE for June 17 using calibration results
600 from June 9 with the same weather conditions is 5.0615% while the MAPE for June 17
601 using calibration results from September 26 which has a different weather condition
602 drastically increased to 14.0573 %. This result also proves that the modeling result is
603 sensitive to the value of the key traffic flow parameters since it was found in the
604 previous section that there is a significant decrease in the value of the said parameters
605 under different weather conditions. It is worth noting that the validation results under
606 the same weather conditions have less accuracy compared to the calibration.
607 Nevertheless, the difference between weather-specific validation processes is very
608 significant. It can be concluded that considering the weather in the validation process
609 of METANET will be more helpful for further practical applications.

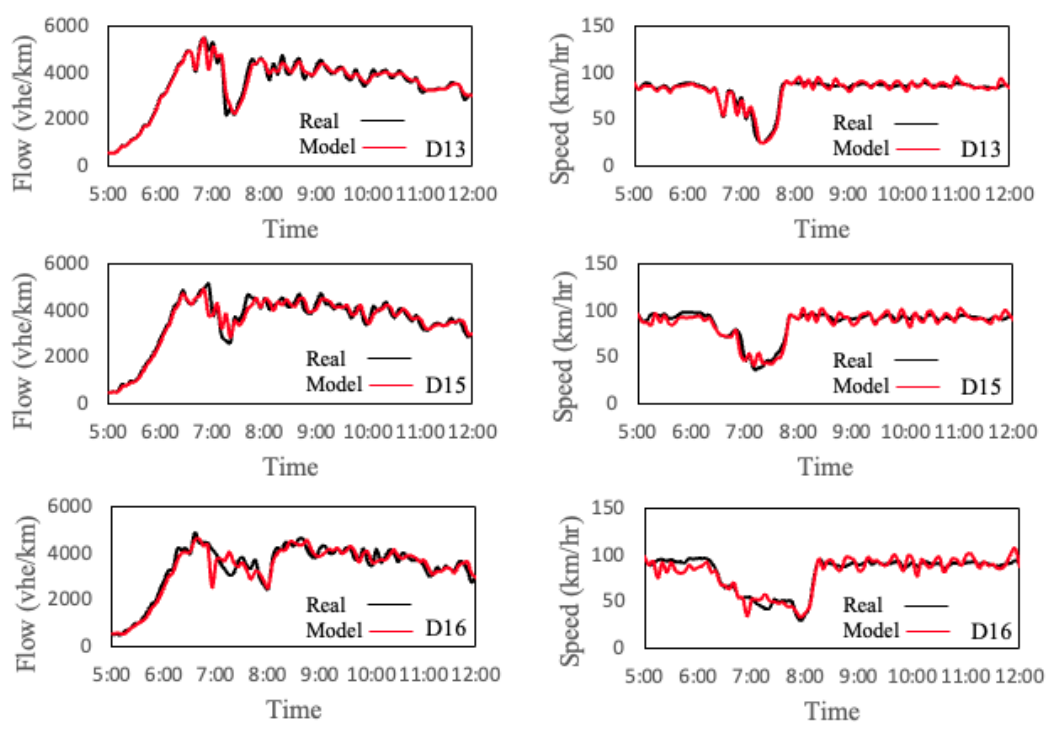
For brevity, only the modeling results on June 17 are shown but the modeling performance for the rest of the days are already shown in Table 3.4. In terms of congestion tracking, weather-specific models were able to sufficiently track the occurrence and dissipation of the congestion as illustrated in Figure 3.7. This is not the case for non-weather-specific modeling depicted in Figure 3.8 which shows that the model was not able to replicate the congestion wave at around 6:30 AM. Figure 3.9 further illustrates the better performance of the weather-specific modeling as shown in the space-time heat maps for both flows and speeds sufficiently predicting the spatiotemporal values compared to non-weather-specific results.

Table 3.4 Summary of results of METANET validation under normal weather conditions in terms of MAPE (%)

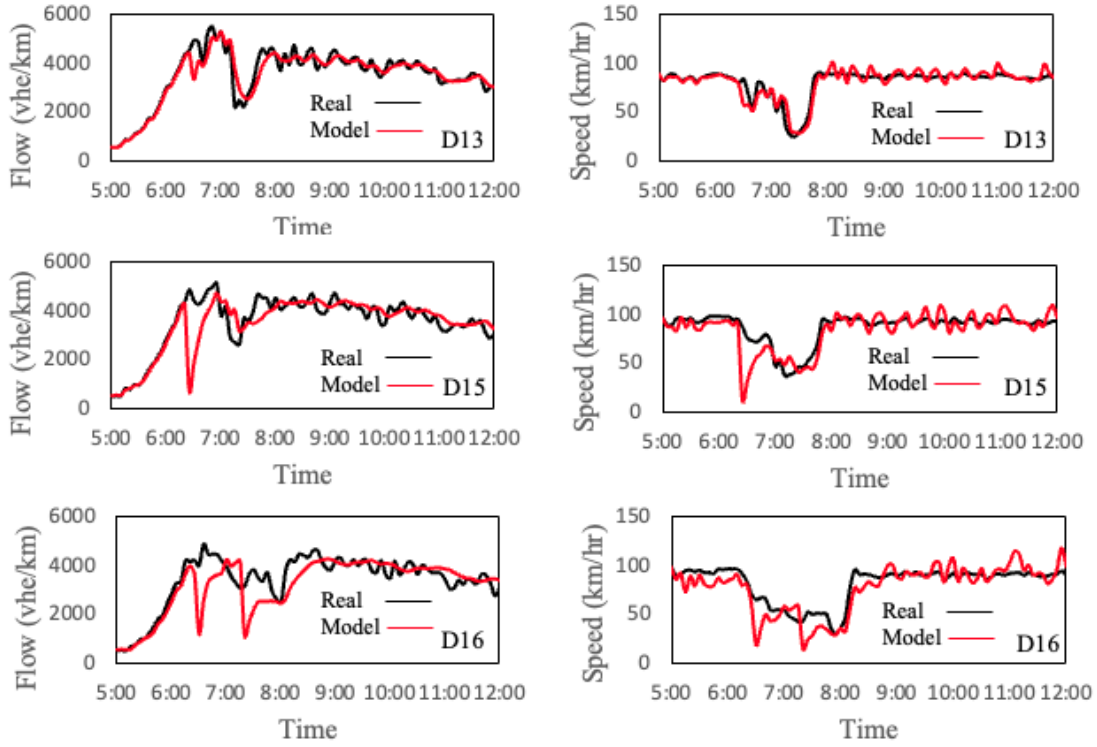
Date	Weather-Specific	Non-Weather-Specific
June 17, 2022	5.0615	14.0573
June 20, 2022	12.4080	15.2733
June 29, 2022	10.8848	11.8386

Figure 3.7

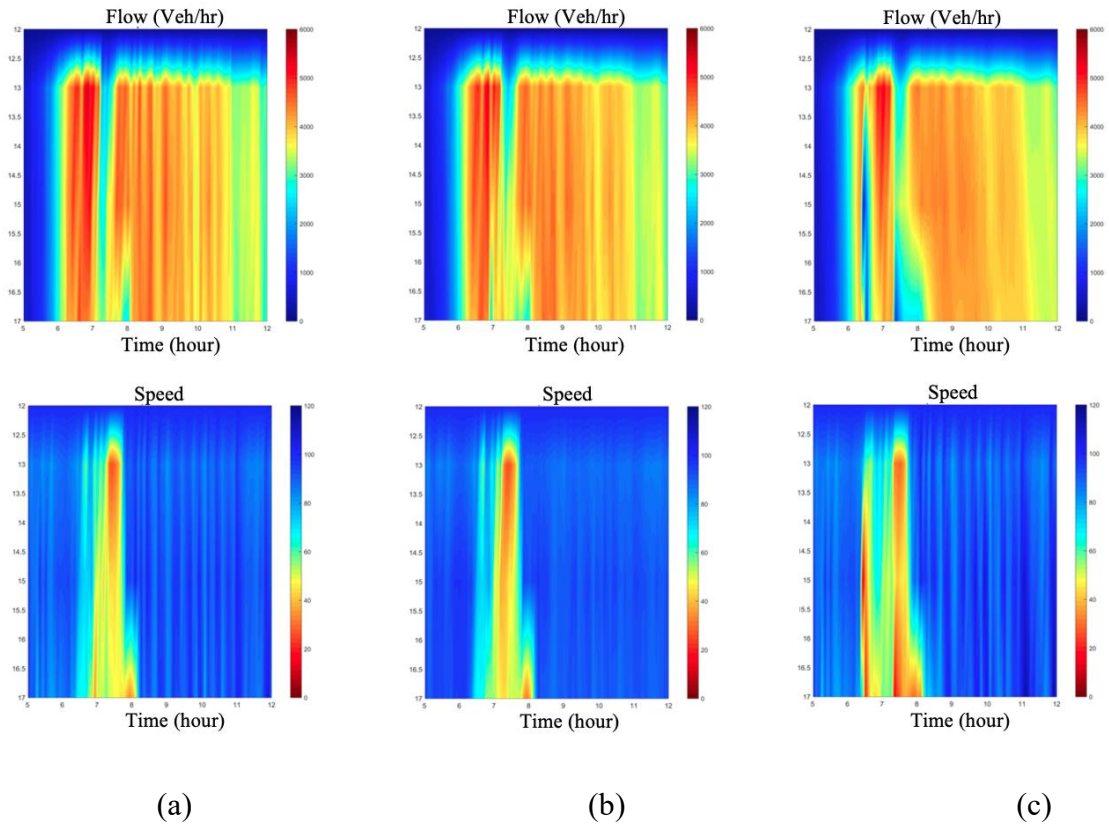
Weather-specific validation of flows on June 17, 2022.



630 **Figure 3.8**
 631
 632 *Non- Weather specific validation of flows on June 17, 2022.*



633 **Figure 3.9**
 634
 635
 636 *Spatiotemporal evolution of flows on June 17, 2022 along the study area (a) real*
 637 *data; (b) weather-specific validation (c) non-weather-specific validation*
 638
 639



3.3.2 Validation of data under bad weather conditions.

Data from June 30, October 6, and October 6 which experienced bad weather conditions, were also validated using the calibration results from September 6 with the same rainy weather conditions. Then, the same data were validated using calibration results from June 9 with normal weather conditions. The performance summary of the validation process is summarized in Table 3.5. Weather-specific means that the model parameters used in the validation are the calibration result under rainy weather conditions on September 26, 2022. On the other hand, non-weather-specific means that the validation model parameters used are those derived under the calibration of a day with normal weather conditions on June 9, 2022. The results show that weather-specific validation always performs better than the non-weather-specific modeling results. This illustrates that weather-specific modeling performs more satisfactorily than non-weather-specific considerations. As an example, the MAPE for June 30 using calibration results from June 9 with the same weather conditions is 3.13% while the MAPE for June 30 using calibration results from September 26 which has a different weather condition drastically increased to 14.27 %.

For brevity, only the modeling results on June 30 are shown but the modeling performance for the rest of the days are already shown in Table 3.5. In terms of congestion tracking, weather-specific models were also able to sufficiently track the occurrence and dissipation of the congestion as illustrated in Figure 3.10 which is not the case for non-weather-specific modeling shown in Figure 3.11 which was not able to reflect the start of the congestion wave. Figures 3.12 further shows the better performance of the weather-specific modeling as shown in the space-time heat maps

for both flows and speeds sufficiently predicting the spatiotemporal values compared to non-weather-specific results.

671

672

673 Table 3.5 Summary of results of METANET validation under bad weather conditions
674 in terms of MAPE (%)

Date	Weather-Specific	Non-Weather-Specific
June 30, 2022	3.13	14.27
October 6, 2022	8.52	11.88
October 7, 2022	7.79	13.05

675

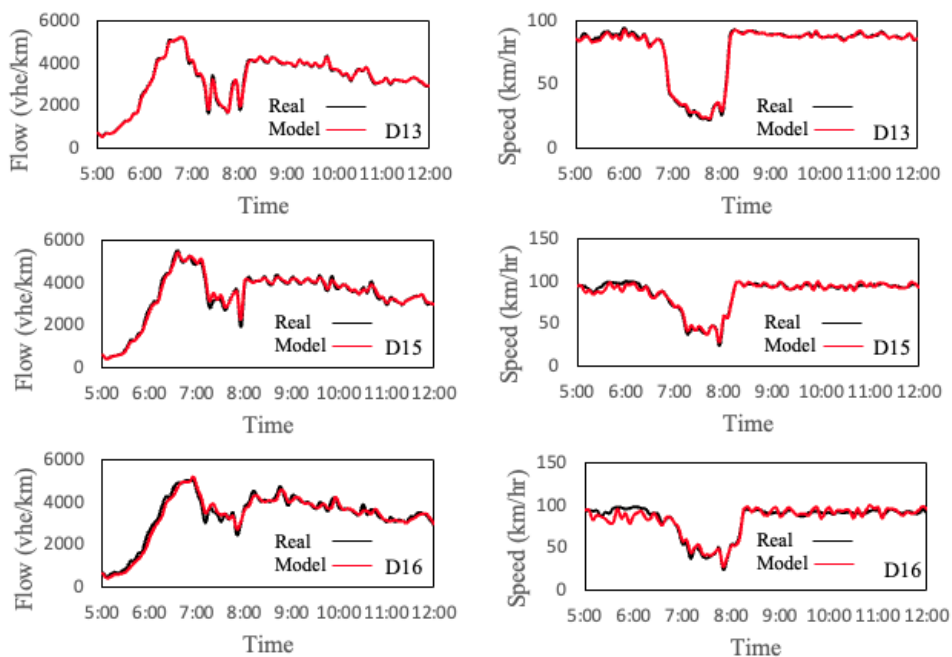
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Figure 3.10

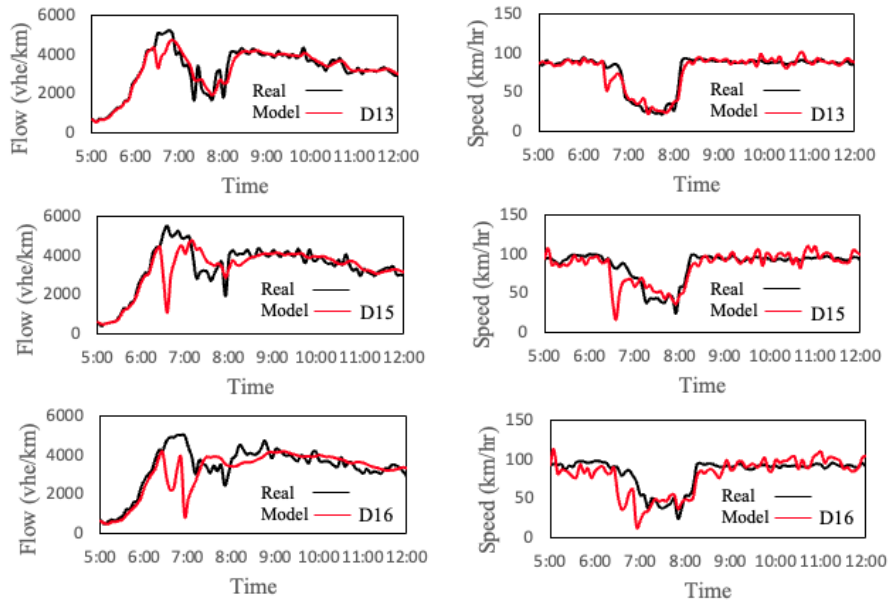
Weather-specific validation of flows on June 30, 2022



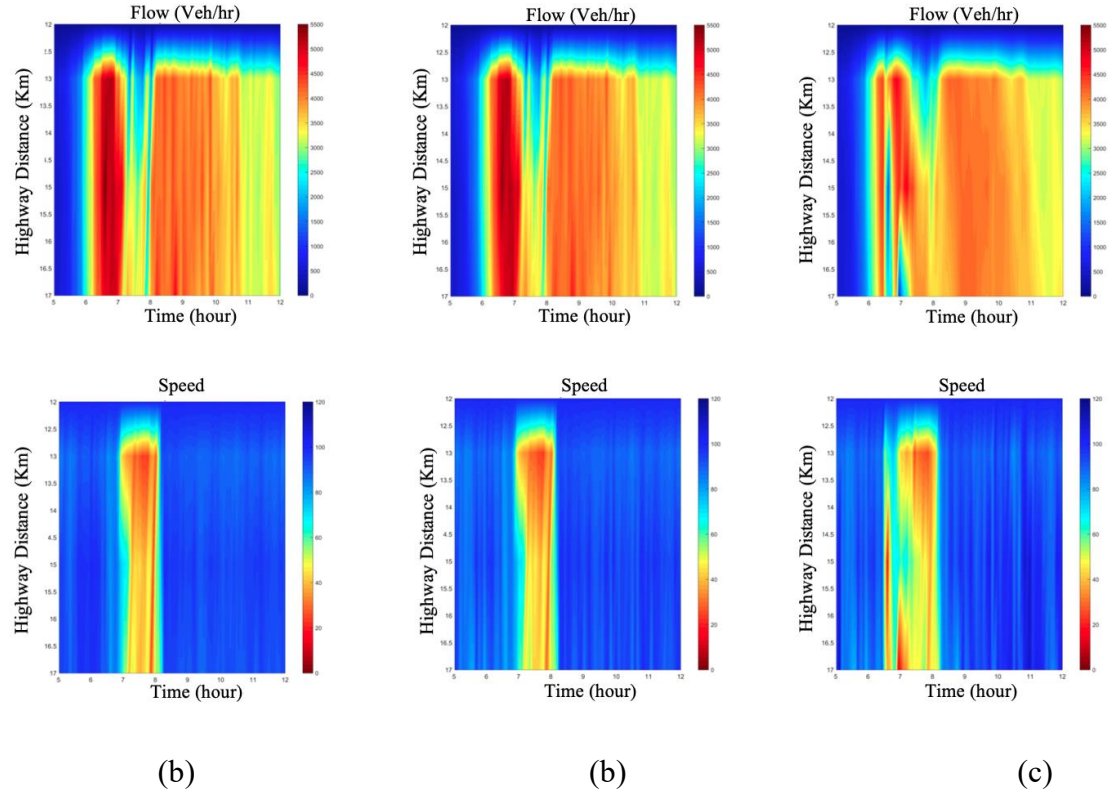
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681 **Figure 3.11**
 682 *Non- Weather specific validation of flows on June 30, 2022*



683
 684
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 686
 687 **Figure 3.12**
 688 *Spatiotemporal evolution of flows on June 30, 2022 along the study area (a) real*
 689 *data; (b) weather-specific validation (c) non-weather-specific validation*
 690



4. CONCLUSIONS AND FUTURE WORK

The study confirmed that the METANET model can accurately simulate traffic dynamics under normal and adverse weather conditions, providing robust calibration and validation results. This capability makes METANET a reliable tool for forecasting traffic behavior in real-world environments, particularly in regions with frequent rainfall. The research further identifies critical parameters such as free-flow speed, capacity, and critical density that significantly influence the model's performance. Adjustments to these parameters enhance METANET's ability to adapt to weather-induced changes in traffic flow, providing a practical foundation for developing more accurate, weather-responsive traffic management systems.

Moreover, METANET demonstrated strong potential in tracking congestion even under varying rainfall intensities. The model effectively captured key weather-induced changes, such as reduced free-flow speed and capacity. These findings validate METANET's adaptability to complex traffic phenomena and show its potential in supporting proactive, data-driven traffic management.

We have replicated the performance of the METANET model in the prediction of traffic states both for calibration and validation as shown in previous studies. To our knowledge, this is the first time that the model was used using data from Bangkok, Thailand. This shows that METANET-based control and operations strategies can apply to the area.

The study indicated that accounting for traffic flow heterogeneity impacts model performance. However, the improvement observed was minimal, likely due to the study focusing on a limited segment of the expressway network and the relatively homogeneous nature of the study area. Future research should incorporate traffic flow heterogeneity or use multiple fundamental diagrams in traffic dynamics modeling. It is anticipated that performance enhancements will be more significant when applied to larger study areas.

The model successfully replicated and tracked congestion patterns under both normal and adverse weather conditions. However, it revealed a notable decrease in key traffic flow parameters, especially free-flow speed and capacity during rainy conditions.

Our findings also highlight the model's sensitivity to key traffic flow parameters, which were observed to change considerably under different weather conditions. Although validation results under the same weather conditions are less accurate compared to calibration performance, the difference in performance between weather-specific and non-weather-specific models is substantial. Therefore, incorporating weather considerations into METANET's validation process enhances its practical applicability.

In terms of model validation, weather-specific modeling consistently outperformed the validation without considering weather factors. Weather-specific models effectively captured both the onset and dissipation of congestion and accurately predicted spatiotemporal values. In contrast, non-weather-specific models failed to replicate the congestion waves.

This study advances the understanding of how weather conditions, particularly rainfall, affect traffic flow parameters and dynamics. By enhancing predictive models for traffic management and control strategies, the research contributes to the development of resilient systems that maintain efficiency and safety during adverse weather. Notably, it is the first comprehensive study to incorporate extensive rainfall data, offering a robust and longitudinal perspective on rainy conditions. This approach addresses gaps in previous research, which either assumed clear weather or analyzed limited rainy-day data, yielding incomplete conclusions.

A significant focus of the study is the rigorous evaluation of the METANET model under varying weather conditions. By assessing its calibration and validation accuracy, sensitivity to parameter variations, and ability to replicate traffic flow characteristics such as congestion tracking, the research highlights the model's adaptability. The innovative use of multiple fundamental diagrams in the calibration process further improves the model's robustness across different traffic regimes and weather scenarios. Additionally, the study includes a practical sensitivity analysis, identifying parameters most affected by adverse weather, enabling traffic engineers to refine models, optimize signal control, enhance route guidance, and develop adaptive traffic management strategies.

The findings have significant practical applications in traffic management and urban planning. They provide insights into designing resilient infrastructure, such as weather-protected lanes, improved drainage systems, and real-time road monitoring technologies. Real-time traffic control systems informed by this research can dynamically adjust speed limits, issue warnings through variable message signs, and respond to rainfall, reducing accidents and managing congestion effectively. Furthermore, the refined METANET parameters improve the predictive accuracy of traffic patterns, aiding resource allocation during adverse weather conditions.

Overall, this study offers a deeper understanding of how weather influences traffic congestion. The findings support the development of weather-responsive strategies that enhance road safety, efficiency, and resilience, ultimately benefiting traffic management, urban planning, and real-time control systems in mitigating the impacts of inclement weather.

The METANET model, while effective for macroscopic traffic flow modeling, has limitations in capturing intricate vehicle interactions in high-density or complex scenarios, such as urban congestion or merging behaviors. Calibration for this study used data from the Burapha Whiti Expressway in Bangkok, making the parameters specific to that roadway and less generalizable to other networks with different geometries, lane configurations, or environmental conditions. The reliance on the Nelder-Mead optimization method may have further limited the findings due to convergence on local minima.

The study analyzed clear and rainy weather but excluded other factors like temperature, wind, or seasonal variations, which could also affect traffic. Low-resolution microwave radar data collected every five minutes may have missed short-term fluctuations, reducing the model's sensitivity to finer changes in traffic flow. Future research could address these gaps by incorporating higher-resolution or additional data sources and exploring a broader range of environmental conditions.

Future studies could also explore integrating METANET with adaptive and predictive control strategies, such as ramp metering, variable speed limits, and coordinated signal timings. These enhancements would improve real-time congestion management and expand the model's applicability to diverse and complex traffic scenarios.

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